Gain-Scheduled H_2 Filter Synthesis via Polynomially Parameter-Dependent Lyapunov Functions with Inexact Scheduling Parameters

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***Abstract*— This paper addresses the design problem of Gain-Scheduled (GS) H_2 filters for Linear Parameter-Varying (LPV) systems under the condition that only inexactly measured scheduling parameters are available. The state-space matrices of the LPV systems are supposed to be polynomially parameter dependent and those of filters which are to be designed are supposed to be rationally parameter dependent. The uncertainties in the measured scheduling parameters are supposed to lie in *a priori* defined convex set. Using structured polynomially Parameter-Dependent Lyapunov Functions (PDLFs), we give a design method of GS H_2 filters, which are robust against the uncertainties in the measured scheduling parameters, in terms of parametrically affine Linear Matrix Inequalities (LMIs). Our proposed method includes robust filter design as a special case. A numerical example demonstrates the effectiveness of our method.**

I. INTRODUCTION

After the proposition of the filter design using Linear Matrix Inequalities (LMIs) in [1], many papers on filter design problem have been reported, e.g. [2]–[12]. Some of them tackle robust filter design problem for Linear Parameter-Varying (LPV) systems or Linear Time-Invariant Parameter-Dependent (LTIPD) systems, and others tackle Gain-Scheduled (GS) filter design problem for LPV systems. Generally speaking, designing filters for LPV/LTIPD systems needs to solve Parameter-Dependent LMIs (PDLMIs). On this issue, several powerful methods, such as, Sum-of-Squares (SOS) approach [13]–[15], matrix dilation or Slack Variable (SV) approach [16], coefficient check approach [17], have been proposed and their effectiveness has also been demonstrated. Thus, in a sense, we can easily design filters for LPV/LTIPD systems with the aid of those methods.

As illustrated in [4], [8], [11], [12], if the scheduling parameters which describe the changes of the plant dynamics are available, it is well known that GS filters have better performance than robust filters. However, existing design methods of GS filters assume that the scheduling parameters are *exactly* measurable and available, which is impossible in real systems. Thus, design methods of GS filters which are robust against the uncertainties in the measured scheduling parameters have been desired. Several papers have already tackled similar problems [18], [19]; however, to author's knowledge, very few results on the design problem for GS filters which exploit inexact scheduling parameters have been reported. In [20], a design method on GS H_∞ filters which exploit inexactly measured scheduling parameters has been successfully proposed. In this note, we show the counterpart result for H_2 filter design problem[1]. Similarly to [20], the uncertainties are supposed to be in *a priori* defined convex set, which can include a hyper-rectangle, and to vary with time. Using structured polynomially Parameter-Dependent Lyapunov Functions (PDLFs), we propose a design method for GS H_2 filters which are robust against the uncertainties in the measured scheduling parameters. We also show that our method encompasses a design method for robust H_2 filters as a special case.

This paper is organized as follows: In section II, we briefly review the conventional design method of GS H_2 filters using Parameter-Dependent Lyapunov Functions (PDLFs), then define our addressed problem. In section III, we show our design method and give some remarks on our method. In section IV, a simple numerical example is introduced to illustrate our results. Finally, we give concluding remarks.

In this note, we use the following notations. $\langle X\rangle$ is the shorthand notation of $X+X^T$, $0_{n,m}$, I_n and $\mathbf{0}$ respectively denote an $n\times m$-dimensional zero matrix, an n-dimensional identity matrix and an appropriately dimensional zero matrix, $\mathcal{R}^{n\times m}$ and $\mathcal{S}^n$ respectively denote sets of $n\times m$-dimensional real matrices and $n\times n$-dimensional symmetric real matrices, $\otimes$ denotes Kronecker product, and $*$ in matrices denotes an abbreviated off-diagonal block. For a symmetric matrix $X=\begin{bmatrix} X_{11} & \cdots & X_{1m} \\ \vdots & \ddots & \vdots \\ X_{m1} & \cdots & X_{mm}\end{bmatrix}\in\mathcal{S}^{nm}$, in which the dimensions of X_{ij} are $n\times n$, $\mathsf{Tr}_n(X)$ denotes $\begin{bmatrix} \mathrm{Tr}(X_{11}) & \cdots & \mathrm{Tr}(X_{1m}) \\ \vdots & \ddots & \vdots \\ \mathrm{Tr}(X_{m1}) & \cdots & \mathrm{Tr}(X_{mm})\end{bmatrix}$.

II. PRELIMINARIES

In this section, we first review the conventional design method of GS H_2 filters using PDLFs, in which change-of-variables [21] is applied, then define our addressed problem.

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[1]Rigorously speaking, H_2 performance cannot be defined in our addressed problem, because as the plant is an LPV system the augmented system with the plant and designed GS filter is also an LPV system. We use this terminology so that readers can easily grasp our addressed problem with admitting that this terminology is slightly abused.

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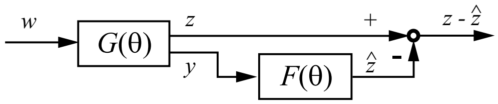

Fig. 1. Block diagram of filter $F(\theta)$ for LPV system $G(\theta)$

A. Conventional Design Method

In this subsection, we briefly review the conventional design method of GS H_2 filters for LPV systems using PDLFs.

Suppose that a stable LPV plant system $G(\theta)$ with k independent scalar parameters $\theta = [\theta_1 \ \cdots \ \theta_k]^T$ is given.

$$G(\theta) : \begin{cases} \dot x = A(\theta)x + B(\theta)w \\ z = C_1(\theta)x \\ y = C_2(\theta)x + D(\theta)w \end{cases}, \tag{1}$$

where $x \in \mathcal{R}^n$ is the state vector with $x = \mathbf{0}$ at $t = 0$, $w \in \mathcal{R}^{n_w}$ is the disturbance input vector, $z \in \mathcal{R}^{n_z}$ is the vector of signals to be estimated, and $y \in \mathcal{R}^{n_y}$ is the vector of measurement outputs. The parameters θ_i, which represent plant uncertainties or the changes of plant dynamics, are supposed to be time-varying (possibly time-invariant).

The ranges of θ_i and $\dot\theta_i$ (the derivative of θ_i with respect to time) are assumed to lie in closed convex sets Ω_θ and Λ_θ, which are known in advance: $\theta(t) \in \Omega_\theta,\ \dot\theta(t) \in \Lambda_\theta,\ \forall t \ge 0$, where $\dot\theta = [\dot\theta_1 \ \cdots \ \dot\theta_k]^T$.

For LPV system (1), we consider a full-order GS filter $F(\theta)$.

$$F(\theta) : \begin{cases} \dot x_f = A_f(\theta)x_f + B_f(\theta)y \\ \hat z \ = C_f(\theta)x_f \end{cases}, \tag{2}$$

where $x_f \in \mathcal{R}^n$ is the state vector with $x_f = \mathbf{0}$ at $t = 0$, $\hat z \in \mathcal{R}^{n_z}$ is the vector of estimated signals of z (see Fig. 1). The state-space matrices $A_f(\theta)$, $B_f(\theta)$ and $C_f(\theta)$, which have appropriate dimensions, are to be designed.

Similarly to [21], the following lemma is easily derived from Lemma 4 in the appendix, which is for the design of GS H_2 output-feedback controllers, with a candidate of PDLFs being set as $x_a^T X_a(\theta)^{-1} x_a$ where $x_a = \left[x^T \ x_f^T\right]^T$.

Lemma 1: For a given positive number γ_2, suppose that there exist continuously differentiable positive definite matrices $R(\theta) \in \mathcal{S}^n$ and $S(\theta) \in \mathcal{S}^n$, a positive definite matrix $N(\theta) \in \mathcal{S}^{n_w}$, and matrices $\mathcal{A}_f(\theta) \in \mathcal{R}^{n\times n}$, $\mathcal{B}_f(\theta) \in \mathcal{R}^{n\times n_y}$ and $\mathcal{C}_f(\theta) \in \mathcal{R}^{n_z\times n}$ such that (3), (4) and (5) hold. Then, a stable GS filter, whose state-space matrices are given as in (7), satisfies (6) for $w = \delta(0)w_0$, where w_0 is a random variable satisfying $\mathcal{E}\left(w_0 w_0^T\right) = I_{n_w}$ and $\delta(\cdot)$ is Dirac's delta function.

$$\begin{bmatrix} N(\theta) & * \\ \begin{bmatrix} B(\theta) \\ S(\theta)B(\theta) + \mathcal{B}_f(\theta)D(\theta) \end{bmatrix} & \begin{bmatrix} R(\theta) & I_n \\ I_n & S(\theta) \end{bmatrix} \end{bmatrix} > 0, \quad \forall\theta \in \Omega_\theta \tag{3}$$

$$\begin{bmatrix} \begin{bmatrix} A(\theta)R(\theta) - \dot R(\theta) & A(\theta) \\ \mathcal{A}_f(\theta) & \begin{pmatrix} \dot S(\theta) + S(\theta)A(\theta) \\ + \mathcal{B}_f(\theta)C_2(\theta) \end{pmatrix} \end{bmatrix} & * \\ \begin{bmatrix} C_1(\theta)R(\theta) - \mathcal{C}_f(\theta) & C_1(\theta) \end{bmatrix} & -I_{n_z} \end{bmatrix} < 0, \quad \forall\left(\theta, \dot\theta\right) \in \Omega_\theta \times \Lambda_\theta \tag{4}$$

$$\gamma_2^2 - \mathrm{Tr}\,(N(\theta)) > 0,\ \forall\theta \in \Omega_\theta \tag{5}$$

$$\sup_{(\theta,\dot\theta)\in\Omega_\theta\times\Lambda_\theta} \mathcal{E}\left(\int_0^\infty (z - \hat z)^T (z - \hat z)dt\right) < \gamma_2^2 \tag{6}$$

$$\begin{cases} A_f(\theta) = N(\theta)^{-1}\Big[\mathcal{A}_f(\theta) - S(\theta)A(\theta)R(\theta) + S(\theta)\dot R(\theta) \\ \qquad + N(\theta)\dot M(\theta)^T - \mathcal{B}_f(\theta)C_2(\theta)R(\theta)\Big] M(\theta)^{-T} \\ B_f(\theta) = N(\theta)^{-1}\mathcal{B}_f(\theta) \\ C_f(\theta) = \mathcal{C}_f(\theta)M(\theta)^{-T} \end{cases}, \tag{7}$$

where matrices $M(\theta), N(\theta) \in \mathcal{R}^{n\times n}$ are arbitrary matrices satisfying $I_n - R(\theta)S(\theta) = M(\theta)N(\theta)^T$.

In this lemma, $X_a(\theta)$ is set as $\Pi_1(\theta)\Pi_2(\theta)^{-1}$ with $\Pi_1(\theta) = \begin{bmatrix} R(\theta) & I_n \\ M(\theta)^T & \mathbf{0} \end{bmatrix}$ and $\Pi_2(\theta) = \begin{bmatrix} I_n & S(\theta) \\ \mathbf{0} & N(\theta)^T \end{bmatrix}$.

Note that the existence of H_2 filters does not depend on the factorization for $M(\theta)$ and $N(\theta)$; that is, without loss of generality, $M(\theta)$ and $N(\theta)$ can be set as follows:

$$N(\theta) = S(\theta) - R(\theta)^{-1},\ M(\theta) = -R(\theta). \tag{8}$$

Then, it is easily confirmed that $P_a(\theta) = X_a(\theta)^{-1}$ is set as

$$P_a(\theta) = \begin{bmatrix} P_1(\theta) & P_2(\theta) \\ P_2(\theta) & P_2(\theta) \end{bmatrix}, \tag{9}$$

where $P_1(\theta) = S(\theta)$ and $P_2(\theta) = S(\theta) - R(\theta)^{-1}$. Thus, without loss of generality, PDLFs can be set as $x_a^T P_a(\theta) x_a$ using $P_a(\theta)$ in (9).

Using $P_a(\theta)$ in (9), the following lemma is directly derived from [22].

Lemma 2: For a given positive number γ_2, suppose that there exist continuously differentiable positive definite matrices $P_1(\theta) \in \mathcal{S}^n$ and $P_2(\theta) \in \mathcal{S}^n$, a positive definite matrix $N(\theta) \in \mathcal{S}^{n_w}$, and matrices $\mathcal{A}_f(\theta) \in \mathcal{R}^{n\times n}$, $\mathcal{B}_f(\theta) \in \mathcal{R}^{n\times n_y}$ and $\mathcal{C}_f(\theta) \in \mathcal{R}^{n_z\times n}$ such that (10), (11), which is at the top of the next page, and (5) hold. Then, a stable GS filter, whose state-space matrices are given as in (12), satisfies (6).

$$\begin{bmatrix} N(\theta) & * \\ \begin{bmatrix} P_1(\theta)B(\theta) + \mathcal{B}_f(\theta)D(\theta) \\ P_2(\theta)B(\theta) + \mathcal{B}_f(\theta)D(\theta) \end{bmatrix} & \begin{bmatrix} P_1(\theta) & P_2(\theta) \\ P_2(\theta) & P_2(\theta) \end{bmatrix} \end{bmatrix} > 0, \quad \forall\theta \in \Omega_\theta \tag{10}$$

$$\begin{cases} A_f(\theta) = P_2(\theta)^{-1}\mathcal{A}_f(\theta) \\ B_f(\theta) = P_2(\theta)^{-1}\mathcal{B}_f(\theta) \\ C_f(\theta) = \mathcal{C}_f(\theta) \end{cases}, \tag{12}$$

Remark 1: Note that the online calculation for the state-space matrices of GS filters designed by Lemma 2 is much simpler than that designed by Lemma 1. Furthermore, GS filters designed by Lemma 2 need no derivatives of parameters in their state-space matrices even if PDLFs are used. On the other hand, Lemma 2 is as little conservative as Lemma 1.

$$\begin{bmatrix} \begin{bmatrix} P_1(\theta)A(\theta)+\mathcal{B}_f(\theta)C_2(\theta)+\dot{P}_1(\theta) & \mathcal{A}_f(\theta)+\dot{P}_2(\theta) \\ P_2(\theta)A(\theta)+\mathcal{B}_f(\theta)C_2(\theta)+\dot{P}_2(\theta) & \mathcal{A}_f(\theta)+\dot{P}_2(\theta) \end{bmatrix} & * \\ \begin{bmatrix} C_1(\theta) & -\mathcal{C}_f(\theta) \end{bmatrix} & -I_{n_z} \end{bmatrix} < 0,\ \forall \left(\theta,\dot{\theta}\right) \in \Omega_\theta \times \Lambda_\theta \tag{11}$$

As shown above, we can design GS H_2 filters with ease. However, these methods assume that the scheduling parameters are *exactly* measurable and available, which is almost impossible in real systems. In other words, the designed filters cannot assure *a priori* defined performance when they are implemented to real world.

B. Problem Definition

In this subsection, we define our addressed problem.

Suppose that the state-space matrices of LPV system (1) are given as follows.

$$\begin{aligned} A(\theta) &= \hat{A}\left(\breve{\theta}\otimes I_n\right), \quad \hat{A} \in \mathcal{R}^{n\times n\sigma} \\ B(\theta) &= \hat{B}\left(\breve{\theta}\otimes I_{n_w}\right),\ \hat{B} \in \mathcal{R}^{n\times n_w\sigma} \\ C_1(\theta) &= \hat{C}_1\left(\breve{\theta}\otimes I_n\right),\ \hat{C}_1 \in \mathcal{R}^{n_z\times n\sigma} \\ C_2(\theta) &= \hat{C}_2\left(\breve{\theta}\otimes I_n\right),\ \hat{C}_2 \in \mathcal{R}^{n_y\times n\sigma} \\ D(\theta) &= \hat{D}\left(\breve{\theta}\otimes I_{n_w}\right),\ \hat{D} \in \mathcal{R}^{n_y\times n_w\sigma} \end{aligned} \tag{13}$$

where

$$\begin{aligned} \breve{\theta}_i^{[m_i]} &= \left[\theta_i^0\ \theta_i^1\ \cdots\ \theta_i^{m_i}\right]^T \in \mathcal{R}^{\sigma_i}, \\ \breve{\theta} &= \breve{\theta}_1^{[m_1]} \otimes \cdots \otimes \breve{\theta}_k^{[m_k]} \in \mathcal{R}^{\sigma}. \end{aligned}$$

Here, $\sigma_i = m_i + 1$ and $\sigma = \sigma(1,k) = \Pi_{i=1}^k \sigma_i$. Both of $\sigma(k+1,k)$ and $\sigma(1,0)$ are defined as 1. The matrices $\hat{A}$, $\hat{B}$, etc. are row-block matrices, each of which represents the coefficient associated with one monomial in $\breve{\theta}$.

The ranges of θ_i and $\dot{\theta}_i$ are assumed to lie in closed convex sets Ω_θ and Λ_θ: $\theta(t) \in \Omega_\theta,\ \dot{\theta}(t) \in \Lambda_\theta,\ \forall t \geq 0$. The vertex sets of Ω_θ and Λ_θ are respectively given as $\mathsf{ver}(\Omega_\theta)$ and $\mathsf{ver}(\Lambda_\theta)$.

Now we define GS H_2 filters which exploit inexactly measured scheduling parameters. It is supposed that the scheduling parameters θ_i are not exactly measured, but measured with some uncertainties δ_i. That is, the i-th scheduling parameter θ_i is assumed to be measured as $\theta_i + \delta_i$.

The range of δ_i and its estimated range of $\dot{\delta}_i$ (the derivative of δ_i with respect to time) are assumed to lie in closed convex sets Ω_δ and Λ_δ, which are known in advance[2]: $\delta(t) \in \Omega_\delta,\ \dot{\delta}(t) \in \Lambda_\delta,\ \forall t \geq 0$, where $\delta = [\delta_1\ \cdots\ \delta_k]^T$ and $\dot{\delta} = [\dot{\delta}_1\ \cdots\ \dot{\delta}_k]^T$. The vertex sets of Ω_δ and Λ_δ are respectively given as $\mathsf{ver}(\Omega_\delta)$ and $\mathsf{ver}(\Lambda_\delta)$.

We now define the filter to be designed as follows.

$$F(\theta,\delta): \begin{cases} \dot{x}_f = A_f(\theta,\delta)x_f + B_f(\theta,\delta)y \\ \hat{z} \ = C_f(\theta,\delta)x_f \end{cases}, \tag{14}$$

where $x_f \in \mathcal{R}^n$ is the state vector with $x_f = \mathbf{0}$ at $t = 0$, and $\hat{z} \in \mathcal{R}^{n_z}$ is the vector of estimated signals of z (see Fig. 2).

[2]It might be very hard to estimate the rate bound for δ_i. However, if it can be estimated with some suitable methods, exploiting such bound reduces conservatism for designing filters.

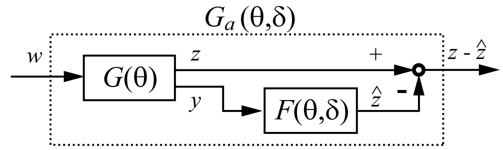

Fig. 2. Block diagram of filter $F(\theta,\delta)$ for LPV system $G(\theta)$

The state-space matrices in (14), which have appropriate dimensions, are to be designed.

If some parameters are assumed to be unmeasurable, i.e. the parameters represent the plant uncertainties, then the state-space matrices of $F(\theta,\delta)$ should be independent of such parameters. Similarly, if a robust filter is to be designed, the state-space matrices in (14) should be all constant. On these issues, we give remarks after showing our design method.

The augmented system $G_a(\theta,\delta)$ comprising $G(\theta)$ and $F(\theta,\delta)$, which is depicted in Fig. 2, is given as follows.

$$G_a(\theta,\delta): \begin{cases} \dot{x}_a \ = A_a(\theta,\delta)x_a + B_a(\theta,\delta)w \\ z - \hat{z} = C_a(\theta,\delta)x_a \end{cases}, \tag{15}$$

where

$$\begin{aligned} A_a(\theta,\delta) &= \begin{bmatrix} A(\theta) & 0_{n,n} \\ B_f(\theta,\delta)C_2(\theta) & A_f(\theta,\delta) \end{bmatrix}, \\ B_a(\theta,\delta) &= \begin{bmatrix} B(\theta) \\ B_f(\theta,\delta)D(\theta) \end{bmatrix}, \\ C_a(\theta,\delta) &= \begin{bmatrix} C_1(\theta) & -C_f(\theta,\delta) \end{bmatrix}. \end{aligned}$$

In this note, we address the following problem.

Problem 1: Suppose that the scheduling parameters θ_i are given as $\theta_i + \delta_i$ with their uncertainties as δ_i. For a given positive number γ_2, find a stable filter $F(\theta,\delta)$ which satisfies (16) for $w = \delta(0)w_0$, where w_0 is a random variable satisfying $\mathcal{E}(w_0 w_0^T) = I_{n_w}$ and $\delta(\cdot)$ is Dirac's delta function.

$$\sup_{(\delta,\dot{\delta})\in\Omega_\delta\times\Lambda_\delta}\ \sup_{(\theta,\dot{\theta})\in\Omega_\theta\times\Lambda_\theta} \mathcal{E}\left(\int_0^\infty (z-\hat{z})^T(z-\hat{z})dt\right) < \gamma_2^2 \tag{16}$$

In sharp contrast to conventional GS filter design methods, e.g. [4], [8], [11], [12], Problem 1 looks for GS H_2 filters which are scheduled by not the real scheduling parameters but the inexactly measured scheduling parameters.

For the system (15), the following lemma is directly derived from the result in [22].

Lemma 3: For a given positive number γ_2, if there exist a continuously differentiable positive definite matrix $P_a(\theta,\delta) \in \mathcal{S}^{2n}$, and a positive definite matrix $N(\theta,\delta) \in \mathcal{S}^{n_w}$ such that (17), (18) and (19) hold, then the system (15) is exponentially stable for all pairs $\left(\theta, \dot{\theta}, \delta, \dot{\delta}\right) \in \Omega_\theta \times \Lambda_\theta \times \Omega_\delta \times \Lambda_\delta$ and satisfies (16).

$$\begin{bmatrix} N(\theta,\delta) & * \\ P_a(\theta,\delta)B_a(\theta,\delta) & P_a(\theta,\delta) \end{bmatrix} > 0,\ \forall(\theta,\delta) \in \Omega_\theta \times \Omega_\delta \quad (17)$$

$$\begin{bmatrix} \langle P_a(\theta,\delta)A_a(\theta,\delta)\rangle + \frac{dP_a(\theta,\delta)}{dt} & * \\ C_a(\theta,\delta) & -I_{n_z} \end{bmatrix} < 0, \quad \forall\left(\theta, \dot{\theta}, \delta, \dot{\delta}\right) \in \Omega_\theta \times \Lambda_\theta \times \Omega_\delta \times \Lambda_\delta \quad (18)$$

$$\gamma_2^2 - \mathrm{Tr}\left(N(\theta,\delta)\right) > 0,\ \forall(\theta,\delta) \in \Omega_\theta \times \Omega_\delta \quad (19)$$

In the next section, we apply Lemma 3 for solving Problem 1.

III. MAIN RESULTS

In this section, we first show our proposed method for Problem 1. Then, we show some extensions of the method in cases that rate bounds for some parameters cannot be estimated and some parameters are not available.

A. Proposed Method

Considering Remark 1, we use PDLFs which are structured similarly to (9).

Before showing our method, we give several definitions.

$$\begin{aligned} e &= [1\ 0_{1,\sigma-1}]^T \in \mathcal{R}^\sigma \\ \eta_i^{[\infty]} &= \begin{bmatrix} I_{m_i} \\ 0_{1,m_i} \end{bmatrix},\ \eta_i^{[0]} = -\begin{bmatrix} 0_{1,m_i} \\ I_{m_i} \end{bmatrix}, \\ \eta_i(\theta_i) &= \theta_i \eta_i^{[\infty]} + \eta_i^{[0]}, \\ \Psi_i^{[\infty]} &= I_{\sigma(1,i-1)} \otimes \eta_i^{[\infty]} \otimes I_{\sigma(i+1,k)} \in \mathcal{R}^{\sigma\times\pi_i}, \\ \Psi_i^{[0]} &= I_{\sigma(1,i-1)} \otimes \eta_i^{[0]} \otimes I_{\sigma(i+1,k)} \in \mathcal{R}^{\sigma\times\pi_i}, \\ \Psi_i(\theta_i) &= \theta_i \Psi_i^{[\infty]} + \Psi_i^{[0]}. \end{aligned}$$

Here, π_i denotes $\sigma m_i/\sigma_i$. Note that $\breve{\theta}^T\Psi_i(\theta_i) = \mathbf{0}$ holds.

A candidate of PDLFs is set as $x_a^T P_S(\theta,\delta) x_a$ with the following $P_S(\theta,\delta)$.

$$P_S(\theta,\delta) = \begin{bmatrix} P(\theta,\delta) & S(\theta,\delta) \\ S(\theta,\delta) & S(\theta,\delta) \end{bmatrix}, \quad (20)$$

where $P(\theta,\delta) \in \mathcal{S}^n$ and $S(\theta,\delta) \in \mathcal{S}^n$.

The parameter-dependency of $P_S(\theta,\delta)$ is set in the sequel. Matrix $P(\theta,\delta)$ is set as follows.

$$\begin{aligned} P(\theta,\delta) &= \left(\breve{\theta} \otimes I_n\right)^T \left(\hat{P} + e \otimes P(\delta)\right), \\ P(\delta) &= \textstyle\sum_{i=1}^k \delta_i P_{\delta_i}, \end{aligned} \quad (21)$$

where $\hat{P} \in \mathcal{R}^{n\sigma\times n}$ is a block column matrix composed of $n\times n$-dimensional symmetric matrices and $P_{\delta_i} \in \mathcal{S}^n$. Noting that $\breve{\theta}^T e = 1$, $P(\theta,\delta)$ is expressed as

$$\tfrac{1}{2}\left(\breve{\theta} \otimes I_n\right)^T \left\langle \hat{P}(e^T \otimes I_n) + \left(ee^T\right) \otimes P(\delta) \right\rangle \left(\breve{\theta} \otimes I_n\right).$$

Matrix $S(\theta,\delta)$ is set as follows.

$$S(\theta,\delta) = S_0 + \sum_{i=1}^k (\theta_i + \delta_i) S_i, \quad (22)$$

where $S_i \in \mathcal{S}^n$ $(i = 0, \cdots k)$. That is, $S(\theta,\delta)$ affinely depends on the inexactly measured scheduling parameters.

As PDLFs are considered, $\frac{dP_a(\theta,\delta)}{dt}$ appears in (18). Accordingly to [20], the term $\frac{d}{dt}P(\theta,\delta)$ is expressed as

$$\left(\breve{\theta} \otimes I_n\right)^T \frac{1}{2}\sum_{i=1}^k \left\langle \Upsilon_i\left(\Xi_i(\dot{\theta}_i), \hat{P}, \dot{\delta}_i P_{\delta_i}\right)\right\rangle \left(\breve{\theta} \otimes I_n\right), \quad (23)$$

where $\Upsilon_i\left(\Xi_i(\dot{\theta}_i), \hat{P}, \dot{\delta}_i P_{\delta_i}\right)$ is defined as

$$\left(I_{\sigma(1,i-1)} \otimes \Xi_i(\dot{\theta}_i) \otimes I_{n\sigma(i+1,k)}\right)^T \hat{P}(e^T \otimes I_n) + \left(ee^T\right) \otimes \left(\dot{\delta}_i P_{\delta_i}\right).$$

with

$$\Xi_i(\dot{\theta}_i) = \dot{\theta}_i \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 2 & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & m_i & 0 \end{bmatrix}.$$

Under these preliminaries, the following theorem is obtained.

Theorem 1: For a given positive number γ_2, suppose that there exist a block column matrix $\hat{P}$ which is composed of $n\times n$-dimensional symmetric matrices, parametrically affine symmetric matrices $P(\delta)$ and $S(\theta,\delta)$, which are respectively defined as in (21) and (22), and $\hat{N}(\delta) \in \mathcal{S}^{n_w\sigma}$, and matrices $\mathcal{A}_f \in \mathcal{R}^{n\times n(k+1)}$, $\mathcal{B}_f \in \mathcal{R}^{n\times n_y(k+1)}$, $\mathcal{C}_f \in \mathcal{R}^{n_z\times n(k+1)}$, $F_i \in \mathcal{R}^{(n_w\pi_i+n\pi_i)\times(n_w\sigma+n\sigma+n)}$, $M_i \in \mathcal{R}^{n\pi_i\times(n\sigma+n+n_z)}$ and $H_i \in \mathcal{R}^{\pi_i\times\sigma}$ such that (24), (25), and (26), the first two of which are at the top of the next page, hold. Then, filter $F(\theta,\delta)$, whose state-space matrices are given as in (27), is stable and satisfies (16).

$$\gamma_2^2\left(ee^T\right) - \mathsf{Tr}_{n_w}\left(\hat{N}(\delta)\right) + \langle\Psi_i(\theta_i)H_i\rangle > 0, \quad \forall(\theta,\delta) \in \mathsf{ver}(\Omega_\theta) \times \mathsf{ver}(\Omega_\delta) \quad (26)$$

$$\left[\begin{array}{c|c} A_f(\theta,\delta) & B_f(\theta,\delta) \\ \hline C_f(\theta,\delta) & \end{array}\right] = \left[\begin{array}{c|c} S(\theta,\delta)^{-1} & \mathbf{0} \\ \hline \mathbf{0} & I_{n_z} \end{array}\right] \left[\begin{array}{c|c} \mathcal{A}_f(\theta,\delta) & \mathcal{B}_f(\theta,\delta) \\ \hline \mathcal{C}_f(\theta,\delta) & \end{array}\right] \quad (27)$$

In (24) and (25), Γ is defined as

$$\Gamma = \begin{bmatrix} \left(\hat{P} + e \otimes P(\delta)\right)\hat{A} + e \otimes \left(\mathcal{B}_f(\theta,\delta)\hat{C}_2\right) & e \otimes \mathcal{A}_f(\theta,\delta) \\ S(\theta,\delta)\hat{A} + \mathcal{B}_f(\theta,\delta)\hat{C}_2 & \mathcal{A}_f(\theta,\delta) \end{bmatrix},$$

and matrices $\left[\begin{array}{c|c} \mathcal{A}_f(\theta,\delta) & \mathcal{B}_f(\theta,\delta) \\ \hline \mathcal{C}_f(\theta,\delta) & \end{array}\right]$ are defined as

$$\left[\begin{array}{c|c} \mathcal{A}_f & \mathcal{B}_f \\ \hline \mathcal{C}_f & \end{array}\right] \mathsf{diag}\left(\begin{bmatrix} 1 \\ \theta+\delta \end{bmatrix}^T \otimes I_n, \begin{bmatrix} 1 \\ \theta+\delta \end{bmatrix}^T \otimes I_{n_y}\right).$$

Proof: Note that all the inequalities (24), (25), and (26) are affine with respect to θ and/or δ. Thus, if they hold at all the vertices of the related parameters, then they hold for all the related parameters.

$$\begin{bmatrix} \hat{N}(\delta) & * \\ \begin{bmatrix} \left(\hat{P} + e \otimes P(\delta)\right) \hat{B} + e \otimes \left(\mathcal{B}_f(\theta,\delta)\hat{D}\right) \\ S(\theta,\delta)\hat{B} + \mathcal{B}_f(\theta,\delta)\hat{D} \end{bmatrix} & \begin{bmatrix} \frac{\langle \hat{P}(e^T \otimes I_n) + (ee^T) \otimes P(\delta)\rangle}{2} & e \otimes S(\theta,\delta) \\ e^T \otimes S(\theta,\delta) & S(\theta,\delta) \end{bmatrix} \end{bmatrix} + \left\langle \sum_{i=1}^{k} \begin{bmatrix} \Psi_i(\theta_i) \otimes I_{n_w} & \mathbf{0} \\ \mathbf{0} & \Psi_i(\theta_i) \otimes I_n \\ \mathbf{0} & \mathbf{0} \end{bmatrix} F_i \right\rangle < 0, \quad \forall\, (\theta,\delta) \in \mathsf{ver}(\Omega_\theta) \times \mathsf{ver}(\Omega_\delta) \tag{24}$$

$$\begin{bmatrix} \left\langle \Gamma + \frac{1}{2}\sum_{i=1}^{k} \begin{bmatrix} \Upsilon_i\left(\Xi_i(\dot{\theta}_i), \hat{P}, \dot{\delta}_i P_{\delta_i}\right) & e \otimes \left((\dot{\theta}_i + \dot{\delta}_i)S_i\right) \\ e^T \otimes \left((\dot{\theta}_i + \dot{\delta}_i)S_i\right) & (\dot{\theta}_i + \dot{\delta}_i)S_i \end{bmatrix} \right\rangle & * \\ \begin{bmatrix} \hat{C}_1 & -\mathcal{C}_f(\theta,\delta) \end{bmatrix} & -I_{n_z} \end{bmatrix} + \left\langle \sum_{i=1}^{k} \begin{bmatrix} \Psi_i(\theta_i) \otimes I_n \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} M_i \right\rangle < 0, \quad \forall \left(\theta, \dot{\theta}, \delta, \dot{\delta}\right) \in \mathsf{ver}(\Omega_\theta) \times \mathsf{ver}(\Lambda_\theta) \times \mathsf{ver}(\Omega_\delta) \times \mathsf{ver}(\Lambda_\delta) \tag{25}$$

Pre- and post multiplications of $\begin{bmatrix} \breve{\theta} \otimes I_{n_w} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \breve{\theta} \otimes I_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_n \end{bmatrix}^T$ and its transpose to (24) from the left and the right respectively lead to (17) with $N(\theta,\delta)$ and $P_a(\theta,\delta)$ being respectively set as $\left(\breve{\theta} \otimes I_{n_w}\right)^T \hat{N}(\delta) \left(\breve{\theta} \otimes I_{n_w}\right)$ and $P_S(\theta,\delta)$ in (20) after conducting the change-of-variable $S(\theta,\delta)B_f(\theta,\delta) = \mathcal{B}_f(\theta,\delta)$. Similarly, pre- and post multiplications of $\begin{bmatrix} \breve{\theta} \otimes I_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_n & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{n_z} \end{bmatrix}^T$ and its transpose to (25) from the left and the right respectively lead to (18) after conducting the change-of-variables $S(\theta,\delta)A_f(\theta,\delta) = \mathcal{A}_f(\theta,\delta)$ and $C_f(\theta,\delta) = \mathcal{C}_f(\theta,\delta)$. Similar algebraic manipulations to (26) lead to (19). This completes the proof. ■

B. Parameter Setting for Decision Matrices

In the sequel, we give several remarks on the parameter setting for decision matrices in Theorem 1.

We first give remarks with respect to the bounds for $\dot{\theta}_i$ and $\dot{\delta}_i$ under the assumption that the i-th scheduling parameter is available. We next give a remark on the availability of the scheduling parameters.

1) Rate Bounds for Parameters: Theorem 1 assumes that the rate bound for δ_i can be estimated. However, it may be very difficult to estimate it. In such case, the rate bound should be set as infinite, i.e. $|\dot{\delta}_i| = \infty$. Consequently, the matrices P_{δ_i} and S_i are set zeros. Then, both $P(\theta,\delta)$ and $S(\theta,\delta)$ are independent of δ_i, which removes the derivatives of δ_i in $\frac{P_a(\theta,\delta)}{dt}$ from (25). However, this modification introduces some conservatism due to the restriction of parameter-dependency for $P(\theta,\delta)$ and $S(\theta,\delta)$. Especially, $S(\theta,\delta)$ is set as independent of the i-th scheduling parameters.

Similarly, if the bound for $\dot{\theta}_i$ cannot be estimated, both $P(\theta,\delta)$ and $S(\theta,\delta)$ are set to be independent of θ_i and δ_i. This modification removes the derivatives of θ_i and δ_i in $\frac{P_a(\theta,\delta)}{dt}$ from (25). In this case, the parameter θ_i can move arbitrarily fast and the state-space matrices in (27) become parametrically affine with respect to the i-th parameter. Furthermore, if all the matrices S_i $(i = 1, \cdots, k)$ are set zeros, then $S(\theta,\delta)$ becomes constant, which reduces online numerical complexity for obtaining the state-space matrices of $F(\theta,\delta)$.

We summarize the above discussions in Table I.

TABLE I
PARAMETER SETTING FOR $P_S(\theta,\delta)$

<table>
<tr><th></th><th>$|\dot{\theta}_i| < \infty$</th><th>$|\dot{\theta}_i| = \infty$</th></tr>
<tr><td>$|\dot{\delta}_i| < \infty$</td><td>Theorem 1</td><td rowspan="2">$P_{\delta_i} = \mathbf{0}$ and all the matrices in $\hat{P}$ related to θ_i are set as $\mathbf{0}$
$S_i = \mathbf{0}$</td></tr>
<tr><td>$|\dot{\delta}_i| = \infty$</td><td>$P_{\delta_i} = \mathbf{0}$
$S_i = \mathbf{0}$</td></tr>
</table>

2) Availability of Parameters: In Theorem 1, all the scheduling parameters are assumed to be available with some uncertainties. However, it may happen that some parameters cannot be available, e.g. the parameter represents the uncertainties of the plant. In such case, if the i-th scheduling parameter cannot be available, then the related matrices in $\mathcal{A}_f$, $\mathcal{B}_f$, $\mathcal{C}_f$, and $S(\theta,\delta)$ are set to be zeros. Consequently, Theorem 1 produces GS filters which are independent of the i-th scheduling parameter.

Similarly, if all the parameters cannot be available, then $P_a(\theta,\delta)$ is set as $\begin{bmatrix} P(\theta) & S \\ S & S \end{bmatrix}$, where $P(\theta)$ has the same definition as $P(\theta,\delta)$ in (21) but all the matrices P_{δ_i} $(i = 1, \cdots, k)$ being set as zeros. This can be set without loss of generality, because the augmented system $G_a(\theta,\delta)$ no longer depends on δ. Similarly, matrix $\hat{N}(\delta)$ can be set as constant.

Then, the following corollary on robust filter design is derived from Theorem 1.

Corollary 1: For a given positive number γ_2, suppose that there exist a block column matrix $\hat{P}$, constant symmetric matrices S_0 and N_0, and matrices $\mathcal{A}_f \in \mathcal{R}^{n\times n}$, $\mathcal{B}_f \in \mathcal{R}^{n\times n_y}$, $\mathcal{C}_f \in \mathcal{R}^{n_z\times n}$, F_i, M_i and H_i such that (24), (25), and (26) in which the followings are set:

$$P(\delta) = 0_{n,n}, \quad S(\theta,\delta) = S_0, \quad \hat{N}(\delta) = N_0, \qquad \delta = \mathbf{0},$$
$$\dot{\delta} = \mathbf{0}, \qquad \mathcal{A}_f(\theta,\delta) = \mathcal{A}_f, \mathcal{B}_f(\theta,\delta) = \mathcal{B}_f, \mathcal{C}_f(\theta,\delta) = \mathcal{C}_f$$

hold. Then, filter $F(\theta,\delta)$ whose state-space matrices are given as $\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & \end{array}\right] = \left[\begin{array}{c|c} S_0^{-1} & \mathbf{0} \\ \hline \mathbf{0} & I_{n_z} \end{array}\right] \left[\begin{array}{c|c} \mathcal{A}_f & \mathcal{B}_f \\ \hline \mathcal{C}_f & \end{array}\right]$ satisfies (16).

IV. NUMERICAL EXAMPLES

We consider Example 2 in [11]. The state-space matrices in (1) is given as

TABLE II
OPTIMAL γ_2 FOR GS H_2 FILTER BY THEOREM 1

ξ	ζ					
	0.01	0.1	1	10	100	10000
0.001	1.374	1.399	1.498	1.591	1.591	1.591
0.1	1.598	1.621	1.671	1.789	1.794	1.794
0.2	1.756	1.780	1.823	1.993	2.027	2.027
0.4	1.993	2.021	2.079	2.363	2.555	2.555
0.5	2.090	2.120	2.183	2.531	2.795	2.795
1	2.389	2.404	2.490	3.162	3.490	3.490
2	2.398	2.407	2.491	3.162	3.490	3.490

TABLE III
OPTIMAL γ_2 FOR ROBUST H_2 FILTER BY COROLLARY 1

ζ	0.01	0.1	1	10	100	10000
γ_2	2.398	2.407	2.491	3.162	3.490	3.490

TABLE IV
MAXIMUM γ_2 FOR $G_a(\theta,\delta)$ USING GS H_2 FILTERS IN TABLE II

ξ	ζ					
	0.01	0.1	1	10	100	10000
0.001	1.349	1.348	1.357	1.433	1.433	1.433
0.1	1.507	1.506	1.517	1.549	1.561	1.561
0.2	1.690	1.675	1.668	1.671	1.695	1.695
0.4	1.907	1.898	1.884	1.864	1.863	1.863
0.5	1.996	1.989	1.971	1.938	1.957	1.958
1	2.281	2.267	2.254	2.218	2.205	2.204
2	2.264	2.263	2.254	2.218	2.192	2.192

TABLE V
MAXIMUM γ_2 FOR $G_a(\theta,\delta)$ USING ROBUST H_2 FILTERS IN TABLE III

ζ	0.01	0.1	1	10	100	10000
γ_2	2.264	2.263	2.254	2.218	2.192	2.192

$$
\begin{aligned}
A(\theta) &= \begin{bmatrix} -0.6 & 4+\theta_1 \\ -4 & -0.6 \end{bmatrix}, \quad B(\theta) = \begin{bmatrix} 0 & 0 \\ 1.5 & 0 \end{bmatrix}, \\
C_1(\theta) &= \begin{bmatrix} 0 & 0.3 \end{bmatrix}, \\
C_2(\theta) &= \begin{bmatrix} 0.3\theta_2 & -1.2 \\ 0.3 & 2-\theta_2 \end{bmatrix}, \quad D(\theta) = \begin{bmatrix} 0.1 & 1.1+\theta_1 \\ 0.1 & 1.1 \end{bmatrix},
\end{aligned}
$$

where scheduling parameter vector is $\theta(= [\theta_1 \ \theta_2]^T)$. The parameter variation for θ is set as $|\theta_i| \le 1$ $(i = 1, 2)$, and their rates are set as $|\dot{\theta}_i| \le \zeta$ $(i = 1, 2)$. The uncertainties for the measured scheduling parameters are set as $|\delta_i| \le \xi$ $(i = 1, 2)$, and their rates are set as $|\dot{\delta}_i| \le 10 \times \zeta$ $(i = 1, 2)$. That is, the rate bounds for the uncertainties in the measured scheduling parameters are set as ten times as those of the real scheduling parameters.

We design GS and robust H_2 filters using our methods with m_i $(i = 1, 2)$ being set as unities for various ζ and ξ. The results for optimized γ_2 are shown in Table II.

Table II indicates that the uncertainties in the measured scheduling parameters heavily affect γ_2 when designing GS H_2 filters. Thus, if the uncertainties are so large, it is better to use robust H_2 filters instead of GS H_2 filters in terms of the online numerical complexity.

For reference, we also design robust H_2 filters using corollary 1. The results are shown in Table III.

To confirm that the designed GS H_2 filters have robustness against the uncertainties in the measured scheduling parameters, we check the maximum H_2 performance for the augmented system $G_a(\theta, \delta)$ with the scheduling parameters for filters being set as $\theta_i \pm \xi$ using the real scheduling parameters θ_i, while the real scheduling parameter is set as $\theta_i = \pm 1$ $(i = 1, 2)$. Thus, totally, 16 combinations, i.e. 4 combinations for θ_i and 4 combinations for δ_i, are considered. The result is shown in Table IV, in which the values of ζ and ξ denote their values when designing GS filters. It is confirmed that the designed GS H_2 filters have robustness against the supposed uncertainties in the measured scheduling parameters at least when the scheduling parameters and their uncertainties are both frozen, i.e. $\dot{\theta}_i = \dot{\delta}_i = 0$.

Similarly, *a posteriori* check for robust filters are conducted and the results are shown in Table V.

V. CONCLUSIONS

This note tackles the design problem of Gain-Scheduled (GS) H_2 filters for Linear Parameter-Varying (LPV) systems whose state-space matrices are polynomially parameter-dependent. Considering that it is almost impossible to obtain the exact values of the scheduling parameters in real systems, it is supposed that the scheduling parameters are measured with some uncertainties which are *a priori* defined. For this practical problem, we give a formulation for GS H_2 filters exploiting inexact scheduling parameters using polynomially Parameter-Dependent Lyapunov Functions (PDLFs) in terms of parametrically affine Linear Matrix Inequalities (LMIs). A numerical example borrowed from the literature supports our results.

APPENDIX

In the sequel, we show a design method of GS H_2 controllers for LPV systems using the method in [21]

Consider the following LPV system

$$
G(\theta) : \begin{cases} \dot{x} = A(\theta)x + B_1(\theta)w + B_2(\theta)u \\ z = C_1(\theta)x + D_{12}(\theta)u \\ y = C_2(\theta)x + D_{21}(\theta)w \end{cases}, \qquad (28)
$$

where $x \in \mathcal{R}^n$ is the state vector with $x = \mathbf{0}$ at $t = 0$, $w \in \mathcal{R}^{n_w}$ is the disturbance input vector, $u \in \mathcal{R}^{n_u}$ is the control input vector, $z \in \mathcal{R}^{n_z}$ is the performance output vector, and $y \in \mathcal{R}^{n_y}$ is the measurement output vector. The parameter vector $\theta = [\theta_1 \ \cdots \ \theta_k]^T$, which denotes the changes of the plant dynamics, are time-varying (possibly time-invariant). The ranges of θ and the derivative of θ are assumed to lie in closed convex sets: $\theta(t) \in \Omega_\theta$, $\dot{\theta}(t) \in \Lambda_\theta$, $\forall t \ge 0$, where $\dot{\theta} = [\dot{\theta}_1 \ \cdots \ \dot{\theta}_k]^T$.

Consider the following full-order GS controller

$$
C(\theta) : \begin{cases} \dot{x}_c = A_c(\theta)x_c + B_c(\theta)y \\ u \ = C_c(\theta)x_c \end{cases}, \qquad (29)
$$

where $x_c \in \mathcal{R}^n$ denotes the state vector with $x_c = \mathbf{0}$ at $t = 0$.

Then, the closed-loop system is given as follows.

$$G_{cl}(\theta) : \begin{cases} \dot{x}_{cl} = A_{cl}(\theta)x_{cl} + B_{cl}(\theta)w \\ z = C_{cl}(\theta)x_{cl} \end{cases}, \tag{30}$$

where $x_{cl} = \left[x^T \ x_c^T\right]^T$, and

$$A_{cl}(\theta) = \begin{bmatrix} A(\theta) & B_2(\theta)C_c(\theta) \\ B_c(\theta)C_2(\theta) & A_c(\theta) \end{bmatrix},$$

$$B_c(\theta) = \begin{bmatrix} B_1(\theta) \\ B_c(\theta)D_{21}(\theta) \end{bmatrix},$$

$$C_c(\theta) = \begin{bmatrix} C_1(\theta) \ D_{12}(\theta)C_c(\theta) \end{bmatrix}.$$

Then, the following lemma is obtained from the result in [22].

Lemma 4: For a given positive number γ_2, if there exist continuously differentiable positive definite matrices $R(\theta) \in \mathcal{S}^n$ and $S(\theta) \in \mathcal{S}^n$, a positive definite matrix $N(\theta) \in \mathcal{S}^{n_w}$, and matrices $\mathcal{A}_c(\theta) \in \mathcal{R}^{n\times n}$, $\mathcal{B}_c(\theta) \in \mathcal{R}^{n\times n_y}$ and $\mathcal{C}_c(\theta) \in \mathcal{R}^{n_u\times n}$ such that (31), (32), which is at the top of the next page, and (33) hold, then GS controller, whose state-space matrices are given as in (35), makes the closed-loop system composed of (28) and (29) exponentially stable for all pairs $\left(\theta, \dot{\theta}\right) \in \Omega_\theta \times \Lambda_\theta$ and satisfies (34).

$$\begin{bmatrix} N(\theta) & * \\ \begin{bmatrix} B_1(\theta) \\ S(\theta)B_1(\theta) + \mathcal{B}_c(\theta)D_{21}(\theta) \end{bmatrix} & \begin{bmatrix} R(\theta) & \mathbf{I} \\ \mathbf{I} & S(\theta) \end{bmatrix} \end{bmatrix} > 0, \quad \forall\theta \in \Omega_\theta \tag{31}$$

$$\gamma_2^2 - \mathrm{Tr}\left(N(\theta)\right) > 0, \ \forall\theta \in \Omega_\theta \tag{33}$$

$$\sup_{(\theta,\dot{\theta})\in\Omega_\theta\times\Lambda_\theta} \mathcal{E}\left(\int_0^\infty z^T z \ dt\right) < \gamma_2^2 \tag{34}$$

$$\begin{cases} A_c(\theta) = N(\theta)^{-1}\Big[\mathcal{A}_c(\theta) - S(\theta)A(\theta)R(\theta) - \\ \qquad \mathcal{B}_c(\theta)C_2(\theta)R(\theta) - S(\theta)B_2(\theta)\mathcal{C}_c(\theta) \\ \qquad + S(\theta)\dot{R}(\theta) + N(\theta)\dot{M}(\theta)^T\Big]M(\theta)^{-T}, \\ B_c(\theta) = N(\theta)^{-1}\hat{B}_c(\theta) \\ C_c(\theta) = \hat{C}_c(\theta)M(\theta)^{-T} \end{cases}, \tag{35}$$

where matrices $M(\theta), N(\theta) \in \mathcal{R}^{n\times n}$ are arbitrary matrices satisfying $I_n - R(\theta)S(\theta) = M(\theta)N(\theta)^T$.

Using the following change-of-variables

$$\begin{aligned} \hat{A}_c(\theta) &= \left[S(\theta)A(\theta) + N(\theta)B_c(\theta)C_2(\theta)\right]R(\theta) \\ &\quad + \left[S(\theta)B_2(\theta)C_c(\theta) + N(\theta)A_c(\theta)\right]M(\theta)^T \\ &\quad - S(\theta)\dot{R}(\theta) - N(\theta)\dot{M}(\theta)^T, \\ \hat{B}_c(\theta) &= N(\theta)B_c(\theta), \\ \hat{C}_c(\theta) &= C_c(\theta)M(\theta)^T, \end{aligned}$$

the lemma is easily proved, similarly to [21]. Thus, the proof is omitted.

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$$\begin{bmatrix} \begin{bmatrix} A(\theta)R(\theta) + B_2(\theta)\mathcal{C}_c(\theta) - \dot{R}(\theta) & A(\theta) \\ \mathcal{A}_c(\theta) & S(\theta)A(\theta) + \mathcal{B}_c(\theta)C_2(\theta) + \dot{S}(\theta) \end{bmatrix} & * \\ \begin{bmatrix} C_1(\theta)R(\theta) + D_{12}(\theta)\mathcal{C}_c(\theta) & C_1(\theta) \end{bmatrix} & -\mathbf{I} \end{bmatrix} < 0,\ \forall \left(\theta, \dot{\theta}\right) \in \Omega_\theta \times \Lambda_\theta \tag{32}$$