Distributed Consensus Under Limited Information

Haopeng Zhang and Qing Hui

***Abstract*— Consensus for networked control systems has a significant application in civil and military applications, while most of the literature focus on the research of consensus for the networked control system with ideal measurements. However, in practice, those assumptions can not be guaranteed properly. Due to the communication link and information storage memory limitations, quantization consensus is more reasonable for the networked control system since the quantized values are less ideal than the perfectly measured values and much more easier to access and transmit in practice. In this paper, we present a novel quantized consensus protocol for the networked control system and prove that near-consensus is achieved under this protocol. To obtain the exact-consensus for the quantized system, a distributed consensus algorithm is further investigated. Finally, the Matlab simulations are provided to verify our theoretical results.**

I. Introduction

Motivated by the common nature behaviors, such as birds and fish which can achieve formations with their information interaction, more and more research focuses on the study of networked behaviors by mimicking the same phenomena exhibited from the nature. Consensus is one of such emerging behaviors for the networked control system, which means that for each agent of the system, with the information interaction with its neighbors, the state of each agent can achieve the same value [1]. On the other hand, near-consensus is a comparably weak definition of consensus, which means the state of each agent goes into the same bounded set [3]. There are so many works investigating the consensus problem for the networked control system with ideal measure condition. For example, the first-order system's consensus property is widely studied in [6], [17], the consensus problem with the second-order system is investigated by [5], [17], and the average consensus is achieved for the position and velocity. Furthermore, instead of studying the linear consensus problem, the nonlinear consensus system attracts more and more attention. In particular, [4] provides a novel stability property for the nonlinear networked control system to achieve consensus, i.e., semistabiliy, unlike asymptotic stability, Lyapunov stability for autonomous dynamical systems does not imply the existence of a continuous Lyapunov function, the semistabiliy does imply the existence of a continuous Lyapunov function.

However, with the limited communication link and the limited storage memory, the dynamical system cannot access, store and process the full information from its neighbors. So the quantized consensus must be taken into account. Quantized consensus by means of gossip algorithms is fully studied in [8], and also the expected value of the time at which the quantized consensus is reached is considered in [9]. [10] investigated the synchronization of passifiable Lurie systems with limited-capacity communication channel, in which an output-feedback control law is proposed. It is shown that the synchronization error exponentially tends to zero. Moreover, [3] developed asymmetrically and symmetrically quantized consensus protocols for the networked control system that involve the only exchange of quantized information between the agents, and guarantee that the closed-loop dynamical network is Lyapounov stable and convergent to a particular set in finite time. This paper extends the results in [3] from continuous-time dynamical systems to discrete-time dynamical systems. By focusing on quantized consensus for discrete-time dynamical systems and iterations, a quantized consensus protocol is presented for the networked control system, and the overall system achieves near-consensus. On the other hand, to obtain the exact-consensus, we present a novel distributed iteration algorithm for the networked control system. Finally, the simulation results are provided to verify our theoretical analysis.

The organization of this paper is as follows: In Section II, the basic knowledge of graph theory and quantizers is provided. The main result of the paper is presented in Section III. In Subsection III-A, the quantized consensus protocol is investigated and the near-consensus is achieved for the networked control system under such a protocol. To obtain the exact-consensus, the quantized distributed iteration algorithm is studied in Subsection III-B. Moreover, the Matlab simulations are provided in Section IV to verify our theoretical analysis. Finally, Section V concludes the paper and the further work is also suggested.

II. Mathematical Preliminaries

A. Graphs

Graph theory is a powerful tool to investigate the networked control systems. In this paper, we use the graph related notation to describe our network model. More specifically, let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ denote an undirected graph with the set of vertices $\mathscr{V} \doteq \{v_1, v_2, v_3, ...\}$ and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ represent the set of edges. The matrix $\mathscr{A}$ with nonnegative adjacency elements $a_{i,j}$ serves as the weighted adjacency matrix. The node index of the $\mathscr{G}$ is denoted as a finite index set $\mathscr{N} = \{1, 2, 3, ...\}$. An edge of $\mathscr{G}$ is denoted by $e_{i,j} = (V_i, V_j)$ and the adjacency elements associated with the edges are positive. We assume $e_{i,j} \in \mathscr{E} \Leftrightarrow a_{i,j} = 1$ and $a_{i,i} = 0$ for all $i \in \mathscr{N}$.

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If there is a path from any node to any other node in the graph, then we call the graph is *connected*. Next, we define the connectivity matrix $\mathcal{C}$ for the graph.

Definition 2.1:

$$\mathcal{C}_{(i,j)} \triangleq \begin{cases} 0, & \text{if } (i,j) \notin \mathscr{E}, \\ 1, & \text{otherwise}, \end{cases} \quad i \neq j, \quad i,j = 1,\ldots,n, \tag{1}$$

$$\mathcal{C}_{(i,i)} \triangleq -\sum_{k=1,\,k\neq i}^{n} \mathcal{C}_{(i,k)}, \quad i = 1,\ldots,n. \tag{2}$$

In this paper, we always assume that the topology of the multi-agent system is connected.

B. quantizer

we use the one-parameter family of quantizers $q_\mu(x) \triangleq \mu q(x/\mu)$, where $\mu > 0$ is an adjustable parameter. In general, $\mu = \mu(t,x)$ can be a function of discrete-time instant $t \in \overline{\mathbb{Z}}_+$ and x. In this research, we only consider the case where μ is a constant.

In this paper, we consider the quantizer with rectangular quantization regions. More specifically, let $q(x)$ be the following form

$$q(x) = \begin{cases} M, & \text{if } x \geq M\Delta, \\ -M, & \text{if } x \leq -M\Delta, \\ \lfloor x/\Delta \rfloor, & \text{if } |x| < M\Delta, \end{cases} \tag{3}$$

where $0 < M \leq \infty$ and if $M < \infty$, then $M \in \mathbb{Z}_+$, and $\lfloor\cdot\rfloor$ denotes the floor function which returns the greatest integer less than or equal to a given real number. Thus, on the interval $(k\Delta,(k+1)\Delta)$ of length Δ, where $k \in \mathbb{Z}$ and $-M \leq k \leq M$, the function q takes on the value k. In this paper, we always assume that $M = \infty$ for simplicity. For two signals x and y, the *inner-signal quantization* is defined to be the error of two quantized signals $q_\mu(x), q_\mu(y)$, that is, $\pm(q_\mu(x) - q_\mu(y))$.

III. Main Result

A. Inner-Signal Quantization Protocol

In this subsection, we consider the following consensus protocol with inner-signal quantization given by

$$\begin{aligned} x_i(t+1) &= x_i(t) + \sum_{j=1,j\neq i}^{n} \mathcal{C}_{(i,j)} K \left[q_\mu(x_j(t)) - q_\mu(x_i(t))\right] \\ &= x_i(t) + \sum_{j=1,j\neq i}^{n} \mathcal{C}_{(i,j)} \mu K \left(\left\lfloor \frac{x_j(t)}{\mu\Delta} \right\rfloor - \left\lfloor \frac{x_i(t)}{\mu\Delta} \right\rfloor\right) \end{aligned}$$

where $t \in \overline{\mathbb{Z}}_+$ is a nonnegative integer, $K > 0$, and we will call (4) *Quantized Consensus System 1*. Furthermore, the near-consensus set for the networked control system is defined as follows:

$$\mathcal{E}_{\mathrm{ss1}} \triangleq \left\{ x \in \overline{\mathbb{R}}_+^n : q_\mu(x_1(t)) = \cdots = q_\mu(x_n(t)) = k \right\} \tag{4}$$

where $\forall k \in \overline{\mathbb{Z}}_+$.

Lemma 3.1: Assume that $\mathcal{C} = \mathcal{C}^{\mathrm{T}}$ and $\operatorname{rank}\mathcal{C} = n-1$. If $0 < K \leq \frac{\Delta}{2}$, then *Quantized Consensus System 1* is positively bounded.

Proof: Consider the nonnegative function $V(x)$ given by

$$V(x) = x^{\mathrm{T}}x. \tag{5}$$

Let $h(x_i) \triangleq \lfloor x_i/\mu\Delta \rfloor$. Then

$$\begin{aligned} & V(x(t+1)/(\mu\Delta)) - V(x(t)/(\mu\Delta)) \\ = \;& 2\sum_{i=1}^{n} x_i(t) \sum_{j=1,j\neq i}^{n} \mathcal{C}_{(i,j)} \frac{K}{\Delta}(h(x_j(t)) - h(x_i(t))) \\ & + \sum_{i=1}^{n} \left[\sum_{j=1,j\neq i}^{n} \mathcal{C}_{(i,j)} \frac{K}{\Delta}(h(x_j(t)) - h(x_i(t))) \right]^2 \\ \leq \;& \frac{2K}{\Delta} \sum_{i=1}^{n} h_i(t) \sum_{j=1,j\neq i}^{n} \mathcal{C}_{(i,j)}[h(x_j(t)) - h(x_i(t))] \\ & + \frac{2K}{\Delta} \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} |h(x_j(t)) - h(x_i(t))| \\ & + \frac{2K^2}{\Delta^2} \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} [h(x_j(t)) - h(x_i(t))]^2 \\ = \;& -\frac{2K}{\Delta} \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} [h(x_j(t)) - h(x_i(t))]^2 \\ & + \frac{2K}{\Delta} \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} |h(x_j(t)) - h(x_i(t))| \\ & + \frac{2K^2}{\Delta^2} \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} [h(x_j(t)) - h(x_i(t))]^2 \\ = \;& \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} \left[-\frac{2K}{\Delta} + \frac{2K^2}{\Delta^2} \right] [h(x_j(t)) - h(x_i(t))]^2 \\ & + \frac{2K}{\Delta} \sum_{i=1}^{n} \sum_{j\in\mathcal{K}_i} |h(x_j(t)) - h(x_i(t))| \end{aligned} \tag{6}$$

where $\mathcal{K}_i \triangleq \mathcal{N}_i \backslash \bigcup_{l=1}^{i-1}\{l\}$ and $\mathcal{N}_i \triangleq \{j \in \{1,\ldots,n\} : \mathcal{C}_{(i,j)} = 1\}$. If $|h(x_j(t)) - h(x_i(t))| = 0$, it is easy to verify that (6) $= 0$, if not, put $|h(x_j(t)) - h(x_i(t))| = s$ with $s > 2$ for $n > 3$, then (6) goes as

$$\left[-\frac{2K}{\Delta} + \frac{2K^2}{\Delta^2} \right] s^2 + \frac{2K}{\Delta} s \tag{7}$$

To satisfy $(6) \leq 0$, then

$$-\frac{2K}{\Delta} + \frac{2K^2}{\Delta^2} \leq 0 \tag{8}$$

$$\left(-\frac{2K}{\Delta} + \frac{2K^2}{\Delta^2} \right) \times 4 + \frac{2K}{\Delta} \times 2 \leq 0 \tag{9}$$

we can get that that $0 < K \leq \frac{\Delta}{2}$. Thus, it follows that *Quantized Consensus System 1* is positively bounded. ■

Lemma 3.2: Consider *Quantized Consensus System 1*. Assume that $\mathcal{C} = \mathcal{C}^{\mathrm{T}}$ and $\operatorname{rank}\mathcal{C} = n-1$. Then for $0 < K \leq \frac{\Delta}{2}$, $x(t) \to \mathcal{E}_{\mathrm{ss}_1}$ as $t \to \infty$.

Proof: Consider the nonnegative function $V(x)$ given by (5). Then it follows from (6) that

$$\begin{aligned} & V(x(t+1)/(\mu\Delta)) - V(x(t)/(\mu\Delta)) \\ \leq \; & \sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}\left[-\frac{2K}{\Delta}+\frac{2K^2}{\Delta^2}\right][h(x_j(t))-h(x_i(t))]^2 \\ & +\frac{2K}{\Delta}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}|h(x_j(t))-h(x_i(t))| \end{aligned} \quad (10)$$

and on the other hand,

$$\begin{aligned} & V(x(t+1)/(\mu\Delta)) - V(x(t)/(\mu\Delta)) \\ \geq \; & \sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}\left[-\frac{2K}{\Delta}+\frac{2K^2}{\Delta^2}\right][h(x_j(t))-h(x_i(t))]^2 \\ & -\frac{2K}{\Delta}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}|h(x_j(t))-h(x_i(t))| \end{aligned} \quad (11)$$

Since, s is an integer, then $s=0$ is the only solution for the equation:

$$\left[-\frac{2K}{\Delta}+\frac{2K^2}{\Delta^2}\right]s^2+\frac{2K}{\Delta}s=0 \quad (12)$$

$$\left[-\frac{2K}{\Delta}+\frac{2K^2}{\Delta^2}\right]s^2-\frac{2K}{\Delta}s=0 \quad (13)$$

Hence, $V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta))=0$ if and only if $h(v_j)=h(v_i)$ for all $i=1,\ldots,n$, $j\in\mathcal{K}_i$. Let $\mathcal{R}\triangleq\{x(t)\in\mathbb{R}^n : V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta))=0\}=\mathcal{R}_1$, where

$$\mathcal{R}_1 \triangleq \{x\in\mathbb{R}^n : h(x_1)=\cdots=h(x_n)\} \quad (14)$$

Clearly, $\mathcal{R}_1=\mathcal{E}_{\rm ss1}$ and $\mathcal{E}_{\rm ss1}$ is an invariant set for (4). Hence, the largest invariant set contained in $\mathcal{R}_1$ is $\mathcal{E}_{\rm ss1}$. Now, it follows from LaSalle invariance principle that $x(t)\to\mathcal{E}_{\rm ss1}$ as $t\to\infty$. ■

Theorem 3.1: Assume that $\mathcal{C}=\mathcal{C}^{\rm T}$ and $\operatorname{rank}\mathcal{C}=n-1$. Furthermore, let $0<K\leq\frac{\Delta}{2}$. Then *Quantized Consensus System 1* achieves near quantized consensus.

This theorem implies that near-consensus is achieved for *Quantized Consensus System 1*, and $x_i\in[k,k+1]$ for all $i=1,\ldots,q$. However, the state variables x_i are not necessarily equal, which is weaker than the standard notion of consensus in the literature.

Instead of the floor function quantizer for *Quantized Consensus System 1*, we can obtain the similar result by adopting the ceiling function quantizer.

Corollary 3.1: Assume that $\mathcal{C}=\mathcal{C}^{\rm T}$ and $\operatorname{rank}\mathcal{C}=n-1$. Furthermore, let $0<K\leq\frac{\Delta}{2}$. Then *Quantized Consensus System 1* can achieve the quantized consensus under the ceiling function quantizer.

Proof: The proof of this corollary is almost identical to that of Theorem 3.1. Therefore, we omit the proof here. ■

B. Distributed Quantized Iteration Algorithm

Alternatively, we consider the consensus protocol with a distributed quantized iteration algorithm given by

$$\begin{aligned} x_i(t+1) &= q_\mu(x_i(t))+\sum\mathcal{C}_{(i,j)}K \\ &\quad [q_\mu(x_j(t))-q_\mu(x_i(t))] \\ &= \mu\left\lfloor\frac{x_i(t)}{\mu\Delta}\right\rfloor+\sum\mathcal{C}_{(i,j)}\mu K \\ &\quad \left(\left\lfloor\frac{x_j(t)}{\mu\Delta}\right\rfloor-\left\lfloor\frac{x_i(t)}{\mu\Delta}\right\rfloor\right) \end{aligned}$$

where $t\in\overline{\mathbb{Z}}_+$ is a nonnegative integer and $K>0$. We call (15) *Quantized Consensus System 2*, and furthermore, define

$$\mathcal{E}_{\rm ss2}\triangleq\left\{x\in\overline{\mathbb{R}}_+^n : x_1/(\mu\Delta)=\cdots=x_n/(\mu\Delta)=k\right\} \quad (15)$$

where $\forall k\in\overline{\mathbb{Z}}_+$, which is the exact-consensus for the networked control system.

Definition 3.1: Quantized Consensus System 2 achieves *exact-quantized consensus* with respect to $\overline{\mathbb{R}}_+^n$ if (15) is positively bounded and $x(t)\to\mathcal{E}_{\rm ss2}$ as $t\to\infty$ for every $x(0)\in\overline{\mathbb{R}}_+^n$.

Lemma 3.3: If K and Δ satisfy

$$\frac{K}{\Delta}\sum_{j=1,j\neq i}^{n}\mathcal{C}_{(i,j)}\leq 1, \quad (16)$$

then for every $x(0)\in\overline{\mathbb{R}}_+^n$, $x(t)\in\overline{\mathbb{R}}_+^n$ for all $t\in\overline{\mathbb{Z}}_+$.

Proof: The result is trivial by mathematical induction. ■

Lemma 3.4: Assume that $\mathcal{C}=\mathcal{C}^{\rm T}$ and $\operatorname{rank}\mathcal{C}=n-1$. Let $n_i\geq 1$ be the number of neighbors of the ith agent in the case where $\mathcal{G}$ is a graph. If $K<\Delta$ and $n_iK\leq\Delta$ for all $i,j=1,\ldots,n$, $i\neq j$, then (15) is positively bounded.

Proof: Consider the nonnegative function $V(x)$ given by

$$V(x)=x^{\rm T}x. \quad (17)$$

Let $h(x_i)\triangleq\lfloor x_i/\mu\Delta\rfloor$. Then

$$\begin{aligned} & V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta)) \\ = \; & 2\sum_{i=1}^{n}h(x_i(t))\sum_{j=1,j\neq i}^{n}\mathcal{C}_{(i,j)}\frac{K}{\Delta}(h(x_j(t))-h(x_i(t))) \\ & +\sum_{i=1}^{n}\left[\sum_{j=1,j\neq i}^{n}\mathcal{C}_{(i,j)}\frac{K}{\Delta}(h(x_j(t))-h(x_i(t)))\right]^2 \\ & +\sum_{i=1}^{n}(h(x_i(t)))^2-\sum_{i=1}^{n}(x_i(t)/(\mu\Delta))^2 \\ \leq \; & 2\sum_{i=1}^{n}h(x_i(t))\sum_{j=1,j\neq i}^{n}\mathcal{C}_{(i,j)}\frac{K}{\Delta}(h(x_j(t))-h(x_i(t))) \\ & +\sum_{i=1}^{n}\left[\sum_{j=1,j\neq i}^{n}\mathcal{C}_{(i,j)}\frac{K}{\Delta}(h(x_j(t))-h(x_i(t)))\right]^2 \end{aligned}$$

$$
\begin{aligned}
&\le -\frac{2K}{\Delta}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}[h(x_j(t))-h(x_i(t))]^2 \\
&\quad +\frac{K^2}{\Delta^2}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}[h(x_j(t))-h(x_i(t))]^2 \\
&= -\frac{2K}{\Delta}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}\left[1-\frac{K}{\Delta}\right][h(x_j(t))-h(x_i(t))]^2 \\
&\le 0 \qquad (18)
\end{aligned}
$$

provided that $K<\Delta$, $n_iK\le\Delta$, $i,j=1,\ldots,n$, $i\neq j$, and $x(0)\in\overline{\mathbb{R}}_+^n$, where $\mathcal{K}_i\triangleq\mathcal{N}_i\backslash\bigcup_{l=1}^{i-1}\{l\}$ and $\mathcal{N}_i\triangleq\{j\in\{1,\ldots,n\}:\mathcal{C}_{(i,j)}=1\}$. Thus, it follows that (15) is positively bounded. ■

Lemma 3.5: Consider *Quantized Consensus System 2*. Assume that $\mathcal{C}=\mathcal{C}^{\mathrm{T}}$ and rank $\mathcal{C}=n-1$. Furthermore, assume $K<\Delta$ and $n_iK\le\Delta$, $i,j=1,\ldots,n$, $i\neq j$. Then $x(t)\to\mathcal{E}_{\mathrm{ss2}}$ as $t\to\infty$ for every $x(0)\in\overline{\mathbb{R}}_+^n$.

Proof: Consider the nonnegative function $V(x)$ given by (17). Then it follows from (18) that $V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta))\le 0$ provided that $K<\Delta$, $n_iK\le\Delta$, $i,j=1,\ldots,n$, $i\neq j$, and $x(0)\in\overline{\mathbb{R}}_+^n$. Next, to show that $x(t)\to\mathcal{E}_{\mathrm{ss2}}$ as $t\to\infty$, note that

$$
\begin{aligned}
&V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta)) \\
\ge\;& -\frac{2K}{\Delta}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}[h(v_j(t))-h(v_i(t))]^2 \\
&+\sum_{i=1}^{n}(h(v_i(t)))^2-\sum_{i=1}^{n}(v_i(t)/(\mu\Delta))^2 \qquad (19)
\end{aligned}
$$

and

$$
\begin{aligned}
&V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta)) \\
\le\;& -\frac{K}{\Delta}\sum_{i=1}^{n}\sum_{j\in\mathcal{K}_i}\left[2-\frac{2K}{\Delta}\right][h(v_j(t))-h(v_i(t))]^2 \\
&+\sum_{i=1}^{n}(h(v_i(t)))^2-\sum_{i=1}^{n}(v_i(t)/(\mu\Delta))^2. \qquad (20)
\end{aligned}
$$

Hence, $V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta))=0$ if and only if $h(v_j)=h(v_i)$ and $h(v_i)=v_i/(\mu\Delta)$ for all $i=1,\ldots,n$, $j\in\mathcal{K}_i$. Let $\mathcal{R}\triangleq\{x(t)\in\mathbb{R}^n: V(x(t+1)/(\mu\Delta))-V(x(t)/(\mu\Delta))=0\}\cap\overline{\mathbb{R}}_+^n=\mathcal{R}_2\cap\mathcal{R}_3\cap\overline{\mathbb{R}}_+^n$, where

$$\mathcal{R}_2 \triangleq \{x\in\mathbb{R}^n: h(x_1)=\cdots=h(x_n)\}, \qquad (21)$$

$$\mathcal{R}_3 \triangleq \bigcap_{i=1}^{n}\{x\in\mathbb{R}^n: h(x_i)=x_i/(\mu\Delta)\}. \qquad (22)$$

Clearly, $\mathcal{R}_2\cap\mathcal{R}_3\cap\overline{\mathbb{R}}_+^n=\mathcal{E}_{\mathrm{ss2}}$ and $\mathcal{E}_{\mathrm{ss2}}$ is an invariant set for *quantized consensus system 2*. Hence, the largest invariant set contained in $\mathcal{R}\cap\overline{\mathbb{R}}_+^n$ is $\mathcal{E}_{\mathrm{ss2}}$. Now, it follows from LaSalle invariance principle that $x(t)\to\mathcal{E}_{\mathrm{ss2}}$ as $t\to\infty$ for every $x(0)\in\overline{\mathbb{R}}_+^n$. ■

Theorem 3.2: Assume that $\mathcal{C}=\mathcal{C}^{\mathrm{T}}$ and rank $\mathcal{C}=n-1$. Furthermore, $0<K<\Delta$. Then *Quantized Consensus System 2* achieves quantized consensus with respect to $\overline{\mathbb{R}}_+^n$. Unlike Theorem 3.1, Theorem 3.2 achieves *exact-consensus*, that is, consensus without error. It is important to note that this difference is due to the different form for *Quantized Consensus System 1* and *Quantized Consensus System 2*. Concretely speaking, *Quantized Consensus System 1* represents a discrete dynamical system with a quantized control input of the form $x_i(k+1)=x_i(k)+u_i(k)$ while *Quantized Consensus System 2* represents a distributed iteration $x_i(k+1)=f(x_i(k))$ with quantized state variables.

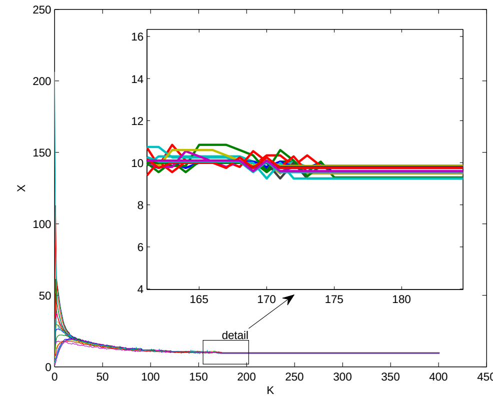

Fig. 1. State evolution under an inner-signal quantized algorithm

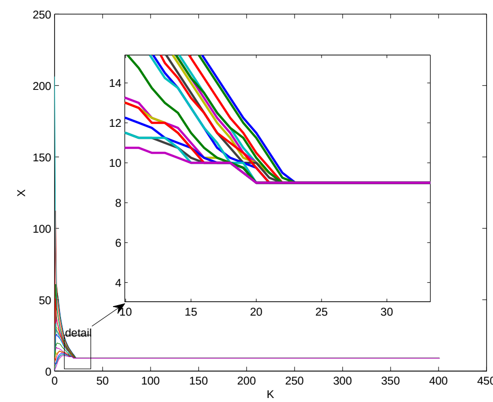

Fig. 2. State evolution under a distributed quantized iteration algorithm

IV. Simulations

In this section, we illustrate some simulation results for the proposed consensus protocols. Given a connected networked control system of 19 agent, Fig.1 shows that the system achieve the near-consensus under the inner-signal quantized algorithm, and from Fig.2, we can conclude that the system achieves consensus via distributed quantized iteration algorithms, which verifies our theoretical results. According to the simulations, we make the following remark.

Remark 4.1: The *Quantized Consensus System 1* (respectively, *Quantized Consensus System 2*) achieves the near-consensus (respectively, exact-consensus) in finite-time.

V. Conclusions and Future Works

In this paper, we study the quantized consensus for the networked control system considering the practical information transmission constraint. Firstly, the inner-signal quantization protocol is provided, and the dynamical system achieves near-consensus which is a weaker notion compared to the exact-consensus. However, to achieve the exact-consensus for the networked control system, a novel distributed quantized iteration algorithm is investigated in the paper. Our further works are focused on the finite-time property of the quantized consensus protocol and the distributed quantized iteration algorithm, which has been displayed by the simulations, also, the quantized consensus for double-integrators is another future work since the fact that the double-integrator is much more precise than the single-integrator to model the practical multi-agent system.

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