Overview of consensus filters for distributed parameter systems utilizing sensor networks

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***Abstract*— This work summarizes two types of consensus filters for a class of distributed parameter systems: consensus and adaptive-consensus filters. Furthermore, it proposes a metric for comparing the disagreement among the spatially local filters. It is assumed that a sensor network consists of groups of sensors, each of which provides a number of state measurements from sensing devices that are not necessarily identical to each other and which only transmit their information to their own sensor group. A metric for examining the disagreement of the local filters, as extended from the finite dimensional case, essentially yields a deterministic analog of the standard deviation of the spatially local filter errors. The disagreement metric is examined for both consensus and adaptive consensus filters. The measure of disagreement is subsequently shown to be linked to the state estimation errors thereby simplifying the performance analysis to simply that of stability of the estimating scheme.**

I. Mathematical Formulation

We are concerned with spatially distributed processes which can mathematically be represented by the following evolution equation

$$\begin{aligned}&\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}_1 w(t) + \mathcal{B}_2 u(t), \quad x(0) = x_0 \in \mathcal{X},\\ &y(t) = \mathcal{C}x(t),\end{aligned} \tag{1}$$

where $x(t)$ denotes the state of the process, $\mathcal{A}$ denotes the state operator and which describes the process dynamics. The operators $\mathcal{B}_1$ and $\mathcal{B}_2$ denote the input operators associated with the process noise and control input. The signals w and u denote the process noise and control signals, respectively. The measured signal $y(t)$ describes the output of the sensing devices and provides information of the state process either in the interior of the spatial domain or at the boundary of the spatial domain, and is an m-dimensional vector. The operator $\mathcal{C}$ describes how the sensing devices operate on and interact with the spatial process. To emphasize the use of m sensing devices, the above system output is rewritten as

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix} = \begin{bmatrix} \mathcal{C}_1 x(t) \\ \mathcal{C}_2 x(t) \\ \vdots \\ \mathcal{C}_m x(t) \end{bmatrix} = \begin{bmatrix} \mathcal{C}_1 \\ \mathcal{C}_2 \\ \vdots \\ \mathcal{C}_m \end{bmatrix} x(t) = \mathcal{C}x(t), \tag{2}$$

where each output operator $\mathcal{C}_i$, $i = 1, 2, \ldots, m$ is associated with the i^{th} sensing device.

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II. Distributed non-adaptive consensus filters

The proposed consensus filters consist of m spatially distributed filters with a disagreement penalty

$$\begin{aligned}\dot{\widehat{x}}_i(t) &= \mathcal{A}_i \widehat{x}_i(t) + \mathcal{B}_2 u(t) + \mathcal{G}_i y_i(t) \\ &\quad + \mathcal{P}_i \textstyle\sum_{j\neq i}^m \kappa_{ij} \left(\widehat{x}_j(t) - \widehat{x}_i(t)\right), \ \widehat{x}_i(0) \in X,\end{aligned} \tag{3}$$

where $\mathcal{G}_i$ denotes the filter gain and is such that each $\mathcal{A}_i \triangleq \mathcal{A} - \mathcal{G}_i\mathcal{C}_i$, $i = 1, \ldots, m$ generates an exponentially stable semigroup on $\mathcal{X}$, $\mathcal{P}_i$ denotes the consensus operator gain and the scalars $\kappa_{ij} > 0$ represent additional weighting of the disagreement terms. The constants $\kappa_{ij} > 0$ can be used to define the connectivity of a given sensor agent to the other agents; if $\kappa_{ij} \neq 0$, $j = 1, \ldots, m$, then the ith agent is connected to all the other agents. In matrix form, they define the topology via the Laplacian [1].

The success of the enforced consensus over the non-consensus case can be realized via the enhanced stability it yields due to the addition of penalty terms. As a continuation of [2], a metric of disagreement among the m consensus filters can be established.

The state errors associated with each of the m consensus filters are given by

$$\begin{aligned}&\dot{e}_i(t) = \mathcal{A}_i e_i(t) + \mathcal{P}_i \textstyle\sum_{j\neq i}^m \kappa_{ij} \left(e_j(t) - e_i(t)\right), \\ &e_i(0) = x(0) - \widehat{x}_i(0) \neq 0.\end{aligned} \tag{4}$$

When the constants κ_{ij} are set to $\kappa_{ij} = \kappa$ which imply an all-to-all connectivity, then the error dynamics simplify further. When the collective dynamics are considered via

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}(t) &= \begin{bmatrix} \mathcal{A}_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \mathcal{A}_m \end{bmatrix} \mathcal{E}(t) \\ &\quad + \kappa \begin{bmatrix} -(m-1)\mathcal{P}_1 & \mathcal{P}_1 & \ldots \\ \vdots & \ddots & \vdots \\ \mathcal{P}_m & \ldots & -(m-1)\mathcal{P}_m \end{bmatrix} \mathcal{E}(t)\end{aligned}$$

they can be compactly written as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}(t) &= (\mathbb{A} + \kappa\mathbb{H})\,\mathcal{E}(t) \\ &= \mathbb{A}_\kappa \mathcal{E}(t)\end{aligned}, \quad \mathcal{E}(0) \in \mathbb{X}, \tag{5}$$

with

$$\mathbb{A} = \begin{bmatrix} \mathcal{A}_1 & \ldots 0 & \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \mathcal{A}_m \end{bmatrix}, \ \mathcal{E}(t) = \begin{bmatrix} e_1(t) \\ \vdots \\ e_m(t) \end{bmatrix},$$

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$$\mathbb{H} = \begin{bmatrix} -(m-1)\mathcal{S}_1^{-1} & \mathcal{S}_1^{-1} & \dots & \mathcal{S}_1^{-1} \\ \mathcal{S}_2^{-1} & -(m-1)\mathcal{S}_2^{-1} & \dots & \mathcal{S}_2^{-1} \\ \vdots & \ddots & \ddots & \vdots \\ \mathcal{S}_m^{-1} & \mathcal{S}_m^{-1} & \dots & -(m-1)\mathcal{S}_m^{-1} \end{bmatrix}$$

where the operators $\mathcal{P}_i$ are assumed invertible with $\mathcal{S}_i = \mathcal{P}_i^{-1}$, $i = 1, \dots, m$.

To gain an insight on the effects and benefits of the consensus terms and subsequent effects on the disagreement terms for the proposed consensus filters, we examine the stability of the collective dynamics in terms of the spectrum of the closed loop operator. Set $\mathbb{S} = \text{diag}\,\{\mathcal{S}_i\}$, $\mathbb{M} = \text{diag}\,\{\mathcal{M}_i\}$ and re-define the inner product on $\mathbb{X}$ by $\langle \phi, \psi \rangle_{\mathbb{X},\mathbb{S}} = \langle \phi, \mathbb{S}\psi \rangle_{\mathbb{X}}$, $\phi, \psi \in \mathbb{X}$. Then

$$\begin{aligned} &\langle \mathbb{A}_\kappa \mathcal{E}(t), \mathcal{E}(t) \rangle_{\mathbb{X},\mathbb{S}} + \langle \mathcal{E}(t), \mathbb{A}_\kappa \mathcal{E}(t) \rangle_{\mathbb{X},\mathbb{S}} \\ &\quad = \langle \mathbb{A}_\kappa \mathcal{E}(t), \mathbb{S}\mathcal{E}(t) \rangle_{\mathbb{X}} + \langle \mathcal{E}(t), \mathbb{S}\mathbb{A}_\kappa \mathcal{E}(t) \rangle_{\mathbb{X}} \\ &\quad = \langle \mathbb{A}\mathcal{E}(t), \mathbb{S}\mathcal{E}(t) \rangle_{\mathbb{X}} + \langle \mathcal{E}(t), \mathbb{S}\mathbb{A}\mathcal{E}(t) \rangle_{\mathbb{X}} \\ &\qquad + \kappa \langle \mathbb{H}\mathcal{E}(t), \mathbb{S}\mathcal{E}(t) \rangle_{\mathbb{X}} + \kappa \langle \mathcal{E}(t), \mathbb{S}\mathbb{H}\mathcal{E}(t) \rangle_{\mathbb{X}}. \end{aligned}$$

Notice that

$$\begin{aligned} &\kappa \langle \mathbb{H}\mathcal{E}(t), \mathcal{E}(t) \rangle_{\mathbb{X},\mathbb{S}} + \kappa \langle \mathcal{E}(t), \mathbb{H}\mathcal{E}(t) \rangle_{\mathbb{X},\mathbb{S}} \\ &\quad = \kappa \langle \mathbb{H}\mathcal{E}(t), \mathbb{S}\mathcal{E}(t) \rangle_{\mathbb{X}} + \kappa \langle \mathcal{E}(t), \mathbb{S}\mathbb{H}\mathcal{E}(t) \rangle_{\mathbb{X}} \\ &\quad = 2\kappa \langle \mathcal{E}(t), \mathbb{J}\mathcal{E}(t) \rangle_{\mathbb{X}} \end{aligned}$$

where the operator $\mathbb{J}$ is defined below in (9). Using the fact that $\langle \mathbb{A}\mathcal{E}(t), \mathbb{S}\mathcal{E}(t) \rangle_{\mathbb{X}} + \langle \mathcal{E}(t), \mathbb{S}\mathbb{A}\mathcal{E}(t) \rangle_{\mathbb{X}} = -\langle \mathcal{E}(t), \mathbb{M}\mathcal{E}(t) \rangle_{\mathbb{X}}$ as a direct consequence of $\mathcal{A}_i^* \mathcal{S}_i + \mathcal{S}_i \mathcal{A}_i = -\mathcal{M}_i$, on $\text{Dom}(\mathcal{A}_i)$, and that the operator $\mathbb{M}$ is coercive with $\langle \mathcal{E}(t), \mathbb{M}\mathcal{E}(t) \rangle_{\mathbb{X}} \geq \varepsilon \|\mathcal{E}(t)\|_{\mathbb{V}}^2$, then enforcing $\mathbb{M}_\kappa \triangleq \mathbb{M} - 2\kappa\mathbb{J}$ be coercive amounts to choosing $\kappa > 0$. The above was derived using

$$0 \leq -2\kappa \langle \phi, \mathbb{J}\phi \rangle_{\mathbb{X}} \leq 2\kappa m \|\phi\|_{\mathbb{V}}^2, \quad \text{for } \phi \in \mathbb{V}.$$

With this choice of κ, we have

$$\begin{aligned} &\langle \mathbb{A}_\kappa \mathcal{E}(t), \mathbb{S}\mathcal{E}(t) \rangle_{\mathbb{X}} + \langle \mathcal{E}(t), \mathbb{S}\mathbb{A}_\kappa \mathcal{E}(t) \rangle_{\mathbb{X}} \\ &\quad = -\langle \mathcal{E}(t), \mathbb{M}\mathcal{E}(t) \rangle_{\mathbb{X}} + 2\kappa \langle \mathcal{E}(t), \mathbb{J}\mathcal{E}(t) \rangle_{\mathbb{X}} \\ &\quad = -\langle \mathcal{E}(t), \mathbb{M}_\kappa \mathcal{E}(t) \rangle_{\mathbb{X}}. \end{aligned}$$

A. Measure of disagreement

Using the expression from [2] to define a measure of disagreement, consider the mean estimate of the m filters given by

$$\mu(t) = \frac{1}{m} \sum_{i=1}^{m} \widehat{x}_i(t). \tag{6}$$

As a measure of disagreement of the m filters, we consider the deviation of each filter state from the mean estimate

$$\delta_i(t) = \widehat{x}_i(t) - \mu(t), \qquad i = 1, 2, \dots, m. \tag{7}$$

To obtain a direct correspondence of the disagreement dynamics and the collective error dynamics (5), the measure of disagreement is rewritten as

$$\begin{aligned} \delta_i(t) &= \widehat{x}_i(t) - \mu(t) \\ &= \tfrac{1}{m}\left(m\widehat{x}_i(t) - \textstyle\sum_{j=1}^m \widehat{x}_j(t) - mx(t) + mx(t) \right) \\ &= \tfrac{1}{m}\left(-me_i(t) + \textstyle\sum_{j=1}^m e_j(t) \right) \\ &= \tfrac{1}{m} \begin{bmatrix} I & \dots & I & -(m-1)I & I & \dots & I \end{bmatrix} \mathcal{E}(t). \end{aligned}$$

Then it is straightforward to see that

$$\delta(t) = \begin{bmatrix} \delta_1(t) \\ \vdots \\ \delta_m(t) \end{bmatrix} = \left(\frac{1}{m} \right) \mathbb{J}\mathcal{E}(t) \tag{8}$$

where

$$\mathbb{J} = \begin{bmatrix} -(m-1)I & I & \dots \\ \vdots & \ddots & \vdots \\ I & \dots & -(m-1)I \end{bmatrix}, \tag{9}$$

with $-m|\mathcal{E}|_{\mathbb{X}}^2 \leq \langle \mathcal{E}, \mathbb{J}\mathcal{E} \rangle_{\mathbb{X}} \leq 0$, and which relates the disagreement dynamics to the collective error dynamics. The dynamics are now given by

$$|\delta(t)|_{\mathbb{X}} \leq |\mathcal{E}(t)|_{\mathbb{X}}, \text{ where } \begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}(t) = \left(\mathbb{A} + \kappa\mathbb{H} \right) \mathcal{E}(t), \\ \mathcal{E}(0) \in \mathbb{X}. \end{cases}$$

One can observe that the convergence rate of $\delta(t)$ is linked to the convergence rate of $\mathcal{E}(t)$. This then leads to the following result

Lemma 1: Given the spatially local consensus filters (3) where each pair $(\mathcal{A}, \mathcal{C}_i)$ satisfies an exponential detectability assumption, then we have that the measure of disagreement of the local filters satisfies

$$|\delta(t)|_{\mathbb{X}} \leq \left(\frac{1}{m} \right) \|\mathbb{J}\| M_{\omega_\kappa} e^{-\omega_\kappa t} |\mathcal{E}(0)|_{\mathbb{X}}, \tag{10}$$

where the constants M_{ω_κ}, ω_κ are associated with the semigroup $\mathbb{T}_\kappa(t)$ generated by $\mathbb{A}_\kappa$ via $\|\mathbb{T}_\kappa(t)\|_{\mathbb{X}} \leq M_{\omega_\kappa} e^{-\omega_\kappa t}$.

III. Distributed adaptive-consensus filters

For the adaptive consensus filters presented below the consensus gain operator is assumed to admit the following parametrization $\mathcal{P}_i = \mathcal{B}_i \Theta \mathcal{C}_i$, where the "input" operator $\mathcal{B}_i$ satisfies certain positive real properties and the $m \times m$ matrix Θ is a matrix of unknown parameters [2]. The goal in this case is to obtain the adaptation of the estimate of the unknown parameter matrix $\widehat{\Theta}(t)$ using Lyapunov redesign methods. The proposed local consensus filters are given by

$$\begin{aligned} \dot{\widehat{x}}_i(t) &= \mathcal{A}_i \widehat{x}_i(t) + \mathcal{B}_2 u(t) + \mathcal{G}_i y_i(t) \\ &\quad + \mathcal{B}_i \Theta_i(t) \mathcal{C}_i \sum_{j \neq i}^{m} \left(\widehat{x}_j(t) - \widehat{x}_i(t) \right), \ \widehat{x}_i(0) \neq x(0). \end{aligned} \tag{11}$$

This artificial positive real-like condition on the operators $\mathcal{B}_i$ is summarized in the form of an assumption below.

Assumption 1 (SPR local filters): Consider each local filter (11), where $\mathcal{A}_i$ is the generator of an exponentially stable

C_0 semigroup on X, $\mathcal{B}_i \in \mathcal{L}(\mathbb{R}^{m_i}, X)$, $\mathcal{C}_i \in \mathcal{L}(X, \mathbb{R}^{m_i})$. Assume that there exists a positive constant μ, $\mathcal{Q}_i \in \mathcal{L}(X)$ and $\mathcal{K}_i \in \mathcal{L}(\mathcal{D}(\mathcal{A}_i), X)$ or $\mathcal{K}_i \in \mathcal{L}(\mathcal{D}(\mathcal{A}_i), \mathbb{R}^{m_i})$ satisfying the constrained Lyapunov equation for $\phi \in \text{Dom}\ (\mathcal{A}_i)$

$$(\mathcal{A}_i + \mu I)^* \mathcal{Q}_i \phi + \mathcal{Q}_i (\mathcal{A}_i + \mu I) \phi = -\mathcal{K}_i^* \mathcal{K}_i \phi \quad \text{(12a)}$$

$$\mathcal{B}_i^* \mathcal{Q}_i \phi = \mathcal{C}_i \phi, \quad i = 1, \ldots, N. \quad \text{(12b)}$$

To extract the adaptation for the matrix $\Theta(t)$, one follows Lyapunov redesign methods. Towards this, the associated local estimation errors are given by

$$\begin{aligned} \dot{e}_i(t) &= \mathcal{A}_i e_i(t) + \mathcal{B}_i \Theta_i(t) \sum_{j \neq i}^{m} \mathcal{C}_i \left(e_j(t) - e_i(t)\right) \\ \varepsilon_i(t) &= \mathcal{C}_i \left(x(t) - \widehat{x}_i(t)\right) = \mathcal{C}_i e_i(t). \end{aligned} \quad \text{(13)}$$

Using the following local Lyapunov functions,

$$V_i(t) = \langle e_i(t), \mathcal{Q}_i e_i(t) \rangle_X + \text{trace}\left(\Theta_i^T(t) \Lambda_i^{-1} \Theta_i(t)\right), \quad (14)$$

where $\mathcal{Q}_i$ is the operator in (2) and $\Lambda_i = \Lambda_i^T > 0$ is an $m_i \times m_i$ adaptation gain matrix [3]. The proposed adaptation is given by

$$\begin{aligned} \dot{\Theta}_i(t) &= -\Lambda_i \varepsilon_i(t) \textstyle\sum_{j \neq i}^{m} \left(\mathcal{C}_i e_j(t) - \mathcal{C}_i e_i(t)\right)^T \\ &= \Lambda_i \varepsilon_i(t) \textstyle\sum_{j \neq i}^{m} \left(\mathcal{C}_i \widehat{x}_j(t) - \mathcal{C}_i \widehat{x}_i(t)\right)^T \end{aligned} \quad \text{(15)}$$

The derivative of the local Lyapunov functions $V_i(t)$ leads to the adaptation rules in (15). By using the adaptation laws to cancel out the coupling terms of the local Lyapunov functions, one then arrives at

$$\begin{aligned} \dot{V}_i(t) &= \langle e_i(t), \left(\mathcal{A}_i^* \mathcal{Q}_i + \mathcal{Q}_i \mathcal{A}_i\right) e_i(t) \rangle_X \\ &= -2\mu_i \langle e_i(t), \mathcal{Q}_i e_i(t) \rangle_X - |\mathcal{K}_i e_i(t)|_X^2 \leq 0. \end{aligned}$$

The remaining stability arguments follow from [2].

To examine the well-posedness of the collective dynamics and provide an expression for the disagreement dynamics, we bring the local error equations into an abstract form

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathcal{E}(t) = \mathbb{A} \mathcal{E}(t) + \left[\begin{array}{ccc} -(m-1)\mathcal{B}_1 \Theta_1 \mathcal{C}_1 & \mathcal{B}_1 \Theta_1 \mathcal{C}_1 & \ldots \\ \vdots & \ddots & \vdots \\ \mathcal{B}_m \Theta_m \mathcal{C}_m & \ldots & -(m-1)\mathcal{B}_m \Theta_m \mathcal{C}_m \end{array} \right. \times \mathcal{E}(t)].$$

The collective error dynamics can now be written

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathcal{E}(t) = \mathbb{A} \mathcal{E}(t) + \left[\begin{array}{ccc} \mathcal{B}_1 \Theta_1(t) & \ldots & \mathcal{B}_m \Theta_m(t) \end{array} \right] \mathbb{I} \mathcal{E}(t)$$

where

$$\mathbb{I} = \left[\begin{array}{c} \mathbb{I}_1 \\ \vdots \\ \mathbb{I}_m \end{array} \right] = \left[\begin{array}{ccc} -(m-1)\mathcal{C}_1 & \ldots & \mathcal{C}_1 \\ \vdots & \ddots & \vdots \\ \mathcal{C}_m & \ldots & -(m-1)\mathcal{C}_m \end{array} \right].$$

Similarly, the dynamics of the gain matrices are given by

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \left[\begin{array}{c} \Theta_1(t) \\ \vdots \\ \Theta_m(t) \end{array} \right] &= - \left[\begin{array}{c} \Lambda_1 \mathcal{C}_1 \\ \vdots \\ \Lambda_m \mathcal{C}_m \end{array} \right] \mathcal{E}(t) \Big(\mathbb{I} \mathcal{E}(t) \Big)^T \\ &= - \left[\begin{array}{ccc} \Lambda_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \Lambda_m \end{array} \right] \mathcal{C} \mathcal{E}(t) \Big(\mathbb{I} \mathcal{E}(t) \Big)^T \end{aligned}$$

To combine the two, we set

$$\Theta(t) = \left[\begin{array}{c} \Theta_1(t) \\ \vdots \\ \Theta_m(t) \end{array} \right], \quad \Lambda = \left[\begin{array}{ccc} \Lambda_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \Lambda_m \end{array} \right],$$

and therefore, we have

$$\left[\begin{array}{c} \mathcal{E}(t) \\ \Theta(t) \end{array} \right]' = \left[\begin{array}{cc} \mathbb{A} & \mathcal{B}\left[\,\cdot\,\right] (\mathbb{I} \mathcal{E}(t)) \\ -\Lambda \mathcal{C}\left[\,\cdot\,\right] (\mathbb{I} \mathcal{E}(t))^T & 0 \end{array} \right] \left[\begin{array}{c} \mathcal{E}(t) \\ \Theta(t) \end{array} \right]$$

This framework is now the same as the one in [4]. Convergence of the collective error dynamics $\lim_{t \to \infty} \|\mathcal{E}\|_{\mathbb{X}} = 0$ can be established, whereas parameter convergence would require the additional assumption of persistence of excitation.

A. Measure of disagreement

Similar to the above, we have $\delta(t) = \left(\frac{1}{m}\right) \mathbb{J} \mathcal{E}(t)$ with $|\delta(t)|_{\mathbb{X}} \leq |\mathcal{E}(t)|_{\mathbb{X}}$ where $\mathcal{E}(0) \in \mathbb{X}$

$$\left[\begin{array}{c} \mathcal{E}(t) \\ \Theta(t) \end{array} \right]' = \left[\begin{array}{cc} \mathbb{A} & \mathcal{B}\left[\,\cdot\,\right] (\mathbb{I} \mathcal{E}(t)) \\ -\Lambda \mathcal{C}\left[\,\cdot\,\right] (\mathbb{I} \mathcal{E}(t))^T & 0 \end{array} \right] \left[\begin{array}{c} \mathcal{E}(t) \\ \Theta(t) \end{array} \right]$$

and which shows the link between the convergence of $\delta(t)$ to that of the collective error $\mathcal{E}(t)$.

IV. Comments on the consensus filters

One of the major differences between the two consensus filters is that each of the coupling terms (operator $\mathbb{H}$ in (5)) for the non-adaptive case was included in the stability arguments of the collective dynamics. The adaptive case had each of the coupling terms be canceled at the level of the local Lyapunov function in order to derive the parameter adaptation.

In both consensus filters, the collective error dynamics converge to zero, thereby indicating that each filter converges to the process state in an appropriate form. Additionally, all filters eventually agree with each other and they all converge to their average value, thereby reaching consensus.

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