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# $H^2$ performance on multiple preview compensation and internal state setting

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***Abstract*— For a generalized $H^2$ preview control problem which includes the command of partial state setting, a design method of the preview control law is clarified. The result stated here covers $H^2$ multiple preview control problems and enables us to derive a compensation law for typical control systems. The feature of the preview compensation law is illustrated with a design example of preview servo-mechanism.**

## I. Introduction

In the design of control systems, the preview of reference signal plays a key role to achieving better performance of the closed loop system and the effect of preview action has been investigated for the improvement of the transient. Recently $H^2$ and $H^\infty$ preview control problems are discussed and the achievable performance is fairly characterized from both state-space and frequency domain approaches [6], [7], [8], [9], [3], [4], [5]. While, from the practical viewpoint of servo-mechanism, simple but effective control strategies have been adopted in the control systems and it is shown that the partial state setting for the internal model (e.g. integrator) enables us to attain favorable transient of the control systems (e.g. [1], [4]).

Motivated by this observation, we focus on a generalized $H^2$ preview control problem and clarify the optimal $H^2$ preview control strategy, which consists of a standard continuous-time input and a command of partial state setting. The problem posed here provides a multiple preview full-information (FI) control law as well as a sequence of partial state setting which attains optimal $H^2$ performance. The $H^2$ preview control problems have been discussed in [7], [4] and, by [7], the $H_2$ multiple preview control law is derived by transforming to a non-standard LQR problem. While [4] deals with the $H^2$ (single) preview problem in the time-domain and it enables to characterizes the $H^2$ performance together with the perturbation of the internal state. In this paper, we characterize the $H^2$ performance of the multiple preview systems and, extending the preliminary result [4], clarify the effect of preview action as well as the partial state setting of the controlled system.

The paper is organized as follows. In Section II, a basic problem for $H^2$ multiple preview compensation problem is solved and the feature of the preview compensation law is discussed. In order to solve the problem, the system response is characterized with integral operators and the optimal control law is constructively given based on completing the square on appropriate function space. In Section III, a generalized problem is solved and the design procedure of the optimal control strategy, which includes a command of partial state-setting, is given based on the characterization of Section II. Typical control problems are discussed in Section IV and the feature of compensation law is investigated with a design example of servo-mechanism (Section V).

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## II. Basic Problem and Preliminary Result

Begin with an $H^2$ preview compensation problem defined as follows:

$$\begin{aligned}\Sigma \quad \dot{x}(t) &= Ax(t) + \sum_{i=0}^{d} \tilde{B}_{1i}\tilde{w}_i(t-h_i) + B_2u(t) \qquad (1)\\ z(t) &= C_1x(t) + D_{12}u(t)\\ y(t) &= \tilde{w}(t), \quad x(0)=0\\ \tilde{w}_i(t) &:= \delta(t)\cdot\tilde{w}_{0i} \;\; (i=0,1,\dots,d)\\ \tilde{w}(t) &:= \begin{bmatrix}\tilde{w}_0^{\mathrm{T}}(t) & \tilde{w}_1^{\mathrm{T}}(t) & \cdots & \tilde{w}_d^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}\\ \tilde{w}_0 &:= \begin{bmatrix}\tilde{w}_{00}^{\mathrm{T}} & \tilde{w}_{01}^{\mathrm{T}} & \cdots & \tilde{w}_{0d}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\end{aligned}$$

where $x(t)\in\mathbb{R}^n$, $\tilde{w}_i(t)\in\mathbb{R}^{\tilde{m}_{1i}}$ $(i=0,1,\dots,d,\ \tilde{m}_1=\sum_{i=0}^{d}\tilde{m}_{1i})$, $u(t)\in\mathbb{R}^{m_2}$, $z(t)\in\mathbb{R}^p$, $y(t)$ are the state, the previewable disturbance, the compensation input, the regulated output, and the measurement respectively. It is noted that the previewable disturbances are described by the delayed terms $\tilde{B}_{1i}\tilde{w}(t-h_i)$ $(i=0,1,\dots,d)$ as the delay-free information $\tilde{w}(t)$ is included in the measurement. The preview times are denoted in ascending order: $0 =: h_0 < h_1 < \cdots < h_d := L$.

For the system $\Sigma$, we make following assumptions:

(A1) $A$ is stable,
(A2) $D_{12}$ has full column rank,

and derive an optimal compensation law which attains $H^2$ optimal performance. A generalized problem, which includes a command of partial state setting, is introduced in Section III.

In order to characterize the preview compensation law for $\Sigma$, we first clarify the relation between the finite-horizon compensation $u\in L_2(0,L;\,\mathbb{R}^{m_2})$ and the functional $J(u,\tilde{w}_0)=\|z\|^2_{L_2(0,\infty;\,\mathbb{R}^p)}$. Since the response $z(t)$ $(t>L)$ is described by the free response of $\Sigma$, the functional

$J(u, \tilde{w}_0)$ is further expressed in the following form:

$$J(u, \tilde{w}_0) = \|z\|^2_{L_2(0,\infty;\,\mathbb{R}^p)} = \left\langle \begin{bmatrix} x(L+) \\ z_{[0,L]} \end{bmatrix}, \mathcal{M} \begin{bmatrix} x(L+) \\ z_{[0,L]} \end{bmatrix} \right\rangle_{\mathbb{R}^n \times L_2(0,\,L;\,\mathbb{R}^p)} \quad (2)$$

$$\begin{bmatrix} x(L+) \\ z_{[0,L]} \end{bmatrix} = \mathcal{F}\tilde{w}_0 + \mathcal{G}u, \quad \mathcal{M} := \begin{bmatrix} M & 0 \\ 0 & \mathcal{I} \end{bmatrix} \quad (3)$$

where $M \geq 0$ is the observability grammian satisfying

$$MA + A^{\mathrm{T}}M + C_1^{\mathrm{T}}C_1 = 0 \quad (4)$$

and $\mathcal{F}$, $\mathcal{G}$ are integral operators defined as follows.

$$\mathcal{F} = \begin{bmatrix} \mathcal{F}_0 \\ \mathcal{F}_1 \end{bmatrix} \in \mathcal{L}(\mathbb{R}^{\tilde{m}_1}, \mathbb{R}^n \times L_2(0,\,L;\,\mathbb{R}^p)) \quad (5)$$

$$\mathcal{F}_0\tilde{w}_0 = \sum_{i=0}^{d} e^{A(L-h_i)}\tilde{B}_{1i}\tilde{w}_{0i}, \quad \forall \tilde{w}_0 \in \mathbb{R}^{\tilde{m}_1}$$

$$(\mathcal{F}_1\tilde{w}_0)(\beta) = \sum_{i=0}^{k} C_1 e^{A(\beta-h_i)}\tilde{B}_{1i}\tilde{w}_{0i}, \quad \forall \tilde{w}_0 \in \mathbb{R}^{\tilde{m}_1}, \quad h_k \leq \beta \leq h_{k+1} \ \ (k = 0, 1, \ldots, d-1)$$

$$\mathcal{G} = \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 \end{bmatrix} \in \mathcal{L}(L_2(0,\,L;\,\mathbb{R}^{m_2}), \mathbb{R}^n \times L_2(0,\,L;\,\mathbb{R}^p)) \quad (6)$$

$$\mathcal{G}_0\phi = \int_0^L e^{A(L-\xi)}B_2\phi(\xi)\,d\xi, \quad \forall \phi \in L_2(0,\,L;\,\mathbb{R}^{m_2})$$

$$(\mathcal{G}_1\phi)(\beta) = \int_0^\beta C_1 e^{A(\beta-\xi)}B_2\phi(\xi)\,d\xi, \ 0 \leq \beta \leq L, \quad \forall \phi \in L_2(0,\,L;\,\mathbb{R}^{m_2})$$

Based on (5),(6), the preview compensation law for (2) is obtained by the following theorem.

*Theorem 1:* Suppose (A1),(A2) hold for the system $\Sigma$. The optimal compensation law $u = u_{\mathrm{opt}} \in L_2(0,\,L;\,\mathbb{R}^{m_2})$ which minimizes (2) is given as follows.

$$u^{\mathrm{opt}}(t) = \sum_{k=0}^{d} K(t, h_k)\tilde{B}_{1k}\tilde{w}_{0k} \quad (7)$$

$$K(\xi,\eta) = \begin{cases} -R^{-1}\begin{bmatrix} D_{12}^{\mathrm{T}}C_1 & B_2^{\mathrm{T}} \end{bmatrix} e^{H\xi}Pe^{-H\eta}\begin{bmatrix} I \\ 0 \end{bmatrix}, & \xi \geq \eta \\ R^{-1}\begin{bmatrix} D_{12}^{\mathrm{T}}C_1 & B_2^{\mathrm{T}} \end{bmatrix} e^{H\xi}P^{\perp}e^{-H\eta}\begin{bmatrix} I \\ 0 \end{bmatrix}, & \xi < \eta \end{cases} \quad (8)$$

$$P = I - \begin{bmatrix} 0 \\ I \end{bmatrix}\left(\begin{bmatrix} -M & I \end{bmatrix} e^{HL}\begin{bmatrix} 0 \\ I \end{bmatrix}\right)^{-1}\begin{bmatrix} -M & I \end{bmatrix} e^{HL} \quad (9)$$

$$P^{\perp} = \begin{bmatrix} 0 \\ I \end{bmatrix}\left(\begin{bmatrix} -M & I \end{bmatrix} e^{HL}\begin{bmatrix} 0 \\ I \end{bmatrix}\right)^{-1}\begin{bmatrix} -M & I \end{bmatrix} e^{HL} \quad (10)$$

$$H = \begin{bmatrix} A & 0 \\ -C_1^{\mathrm{T}}C_1 & -A^{\mathrm{T}} \end{bmatrix} - \begin{bmatrix} B_2 \\ C_1^{\mathrm{T}}D_{12} \end{bmatrix} R^{-1}\begin{bmatrix} D_{12}^{\mathrm{T}}C_1 & B_2^{\mathrm{T}} \end{bmatrix} \quad (11)$$

$$R = D_{12}^{\mathrm{T}}D_{12} > 0 \quad (12)$$

Furthermore the optimal cost of (2) is expressed as follows.

$$J^{\mathrm{opt}}(\tilde{w}_0) := J(u^{\mathrm{opt}}, \tilde{w}_0) = \tilde{w}_0^{\mathrm{T}}\tilde{B}_1^{\mathrm{eT}}X\tilde{B}_1^{\mathrm{e}}\tilde{w}_0 \quad (13)$$

$$\tilde{B}_1^{\mathrm{e}} := \text{block diag}(\tilde{B}_{10}, \tilde{B}_{11}, \ldots, \tilde{B}_{1d}) \quad (14)$$

$$X := [X_{ij}] \in \mathbb{R}^{(d+1)n\times(d+1)n} \quad (15)$$

$$X_{ij} = \begin{cases} \begin{bmatrix} 0 & I \end{bmatrix} e^{Hh_i}Pe^{-Hh_j}\begin{bmatrix} 0 \\ I \end{bmatrix}, & i \geq j \\ \begin{bmatrix} 0 & I \end{bmatrix} e^{Hh_i}P^{\perp}e^{-Hh_j}\begin{bmatrix} 0 \\ I \end{bmatrix}, & i < j \\ (i,j = 0, 1, \ldots, d) \end{cases} \quad (16)$$

■

For the proof of Theorem 1, we first provide a preliminary result on the operators (5),(6).

*Lemma 2:* Under the assumptions (A1),(A2), the following statements hold.

(a) The operator

$$\Delta = \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 + D_{12}\cdot\mathcal{I} \end{bmatrix}^* \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 + D_{12}\cdot\mathcal{I} \end{bmatrix} \quad (17)$$

has a bounded inverse on $L_2(0, L;\, \mathbb{R}^{m_2})$.

(b) The matrix

$$\begin{bmatrix} -M & I \end{bmatrix} e^{HL}\begin{bmatrix} 0 \\ I \end{bmatrix} \quad (18)$$

is nonsingular. ■

*Proof:* (a): Since $\Delta$ is given by sum of the scaled identity operator $D_{12}^{\mathrm{T}}D_{12}\cdot\mathcal{I}$ and the other compact operators, $\Delta$ has a bounded inverse iff it does not have any eigenvalue at origin. We verify by contradiction that $\Delta$ is invertible.

Suppose that $\Delta$ has an eigenvalue $0$ and

$$\langle v, \Delta v\rangle_{L_2(0,L;\,\mathbb{R}^{m_2})} = \left\| \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 + D_{12}\cdot\mathcal{I} \end{bmatrix} v \right\|^2_{L_2(0,L;\,\mathbb{R}^{m_2})} = 0 \quad (19)$$

holds for $v \neq 0$. Introducing an auxiliary variable

$$\tilde{p}(\beta) = \int_0^\beta e^{A(\beta-\xi)}B_2 v(\xi)\,d\xi \quad (20)$$

for $\mathcal{G}_0 v = 0$ and $(\mathcal{G}_1 + D_{12}\cdot\mathcal{I})v = 0$, we have the following equalities.

$$\tilde{p}'(\beta) = A\tilde{p}(\beta) + B_2 v(\beta), \quad 0 \leq \beta \leq L \quad (21)$$

$$\tilde{p}(0) = 0 \quad (22)$$

$$C_1\tilde{p}(\beta) + D_{12}v(\beta) = 0 \quad (23)$$

Furthermore (21)-(23) yield a differential equation

$$\tilde{p}'(\beta) = (A - R^{-1}D_{12}^{\mathrm{T}}C_1)p(\beta), \quad \tilde{p}(0) = 0 \quad (24)$$

and $p = 0$, $v = 0$ are derived from (24),(23). Since this fact contradicts the existence of $v \neq 0$, (a) is proved.

(a)⇔(b): By contraposition, we will show that $\Delta$ has an eigenvalue at origin iff the matrix (18) is singular. Suppose $\Delta v = 0$ or equivalently

$$f^0 = \mathcal{G}_0 v \quad (25)$$

$$f^1 = \mathcal{G}_1 v + D_{12}\cdot v \quad (26)$$

$$\mathcal{G}_0^* f^0 + \mathcal{G}_1^* f^1 + D_{12}^{\mathrm{T}}f^1 = 0 \quad (27)$$

hold for $v \neq 0$, $f = (f^0, f^1)$. Introducing the auxiliary variables (20) and

$$q(\xi) = e^{A^{\mathrm{T}}(L-\xi)} F^{\mathrm{T}} f^0 + \int_{\xi}^{L} e^{A^{\mathrm{T}}(\beta-\xi)} C_1^{\mathrm{T}} f^1(\beta)\, d\beta, \quad (28)$$

the equalities

$$\begin{bmatrix} \tilde{p}'(\beta) \\ q'(\beta) \end{bmatrix} = H \begin{bmatrix} \tilde{p}(\beta) \\ q(\beta) \end{bmatrix}, \quad 0 \le \beta \le L \quad (29)$$

$$\tilde{p}(0) = 0 \quad (30)$$

$$\begin{bmatrix} -M & I \end{bmatrix} \begin{bmatrix} \tilde{p}(L) \\ q(L) \end{bmatrix} = 0 \quad (31)$$

$$v = -R^{-1} \begin{bmatrix} D_{12}^{\mathrm{T}} C_1 & B_2^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \tilde{p} \\ q \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} -M & I \end{bmatrix} e^{HL} \begin{bmatrix} 0 \\ I \end{bmatrix} q(0) = 0 \quad (33)$$

are obtained from (25)-(27).

If $q(0) = 0$, (29),(30),(32) yield $\tilde{p} = 0$, $q = 0$, and $v = 0$. In other words, the matrix (18) must be singular if $\Delta$ has an eigenvalue $0$. Conversely, if $v = 0$, (25),(26) and (20),(28) derive $\tilde{p} = 0$, $q = 0$ i.e. $q(0) = 0$.

By contraposition, it is proved that (a),(b) are equivalent. ■

*Proof of Theorem 1:* Completing the square in (2), we have the equalities:

$$\begin{aligned} J(u, \tilde{w}_0) &= J(u^{\mathrm{opt}}, \tilde{w}_0) \\ &\quad + \langle (u - u_{\mathrm{opt}}), \Delta(u - u_{\mathrm{opt}}) \rangle_{L_2(0,L;\mathbb{R}^{m_2})} \quad (34) \\ u_{\mathrm{opt}} &= -\Delta^{-1} \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 + D_{12} \cdot \mathcal{I} \end{bmatrix}^* \begin{bmatrix} \mathcal{F}_0 \\ \mathcal{F}_1 \end{bmatrix} \tilde{w}_0 \quad (35) \\ J(u^{\mathrm{opt}}, \tilde{w}_0) &= \tilde{w}_0^{\mathrm{T}} \Xi \tilde{w}_0 \quad (36) \\ \Xi &= \begin{bmatrix} \mathcal{F}_0 \\ \mathcal{F}_1 \end{bmatrix}^* \left\{ \mathcal{I} - \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 + D_{12} \cdot \mathcal{I} \end{bmatrix} \Delta^{-1} \right. \\ &\quad \left. \times \begin{bmatrix} \mathcal{G}_0 \\ \mathcal{G}_1 + D_{12} \cdot \mathcal{I} \end{bmatrix}^* \right\} \begin{bmatrix} \mathcal{F}_0 \\ \mathcal{F}_1 \end{bmatrix} \quad (37) \end{aligned}$$

where $\Delta$ is defined by (17).

We first derive the compensation law (7) from (35). Rewriting the equality (35) by

$$f^0 = \mathcal{F}_0 \tilde{w}_0 + \mathcal{G}_0 u_{\mathrm{opt}}, \quad (38)$$

$$f^1 = \mathcal{F}_1 \tilde{w}_0 + \mathcal{G}_1 u_{\mathrm{opt}} + D_{12} u_{\mathrm{opt}}, \quad (39)$$

$$\mathcal{G}_0^* f^0 + \mathcal{G}_1^* f^1 + D_{12}^{\mathrm{T}} f^1 = 0, \quad (40)$$

then introducing the auxiliary variables:

$$p_j(\beta) = \sum_{i=0}^{j-1} e^{A(\beta-h_i)} \tilde{B}_{1i} \tilde{w}_{0i} + \int_0^{\beta} e^{A(\beta-\xi)} B_2 u_{\mathrm{opt}}(\xi)\, d\xi$$
$$h_{j-1} \le \beta \le h_j \ \ (j = 1, 2, \ldots, d) \quad (41)$$

and (28), the following equalities are obtained.

$$u_{\mathrm{opt}}(\beta) = -R^{-1} \begin{bmatrix} D_{12}^{\mathrm{T}} C_1 & B_2^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} p_j(\beta) \\ q(\beta) \end{bmatrix} \quad (42)$$

$$\begin{bmatrix} p_j'(\beta) \\ q'(\beta) \end{bmatrix} = H \begin{bmatrix} p_j(\beta) \\ q(\beta) \end{bmatrix}, \quad \begin{matrix} h_{j-1} \le \beta \le h_j \\ (j = 1, 2, \ldots, d) \end{matrix} \quad (43)$$

$$p_1(0) = \tilde{B}_{10} \tilde{w}_{00} \quad (44)$$

$$p_{j+1}(h_j) = p_j(h_j) + \tilde{w}_{0j} \ \ (j = 1, 2, \ldots, d-1) \quad (45)$$

$$\begin{bmatrix} -M & I \end{bmatrix} \begin{bmatrix} p_d(L) + \tilde{B}_{1d} \tilde{w}_{0d} \\ q(L) \end{bmatrix} = 0 \quad (46)$$

Furthermore the equalities

$$\begin{bmatrix} p_j(\beta) \\ q(\beta) \end{bmatrix} = e^{H\beta} \begin{bmatrix} 0 \\ I \end{bmatrix} q(0) + \sum_{k=0}^{j-1} e^{H(\beta-h_k)} \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{B}_{1k} \tilde{w}_{0k},$$
$$h_{j-1} \le \beta \le h_j \ \ (j = 1, 2, \ldots, d), \quad (47)$$

$$\begin{aligned} \begin{bmatrix} p_d(L) + \tilde{B}_{1d} \tilde{w}_{0d} \\ q(L) \end{bmatrix} &= e^{HL} \begin{bmatrix} 0 \\ I \end{bmatrix} q(0) \\ &\quad + \sum_{k=0}^{d} e^{H(L-h_k)} \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{B}_{1k} \tilde{w}_{0k}, \quad (48) \end{aligned}$$

$$\begin{aligned} 0 &= \begin{bmatrix} -M & I \end{bmatrix} e^{HL} \begin{bmatrix} 0 \\ I \end{bmatrix} q(0) \\ &\quad + \sum_{k=0}^{d} e^{H(L-h_k)} \begin{bmatrix} -M & I \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{B}_{1k} \tilde{w}_{0k}. \quad (49) \end{aligned}$$

follows from (43)-(46). Hence, by Lemma 2, the optimal compensation law is given by (7).

Next, we derive (13). By (37), the matrix $\Xi$ satisfies (25),(26) and

$$\Xi \tilde{w}_0 = \mathcal{F}_0^* f^0 + \mathcal{F}_1^* f^1. \quad (50)$$

Employing the variables (41),(28), the equality

$$\Xi \tilde{w}_0 = \begin{bmatrix} \tilde{B}_{10}^{\mathrm{T}} q(h_0) \\ \tilde{B}_{11}^{\mathrm{T}} q(h_1) \\ \vdots \\ \tilde{B}_{1d}^{\mathrm{T}} q(h_d) \end{bmatrix} \quad (51)$$

follows from (50) and further,

$$\begin{aligned} q(h_i) &= \begin{bmatrix} 0 & I \end{bmatrix} e^{Hh_i} \begin{bmatrix} 0 \\ I \end{bmatrix} q(0) \\ &\quad + \sum_{k=0}^{i} \begin{bmatrix} 0 & I \end{bmatrix} e^{H(h_i-h_j)} \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{B}_{1j} \tilde{w}_{0j}, \quad i = 0, 1, \ldots, d \end{aligned} \quad (52)$$

is obtained by (47). Thus (13) is derived from (51),(52). ■

*Remark 3:* The matrices $P$, $P^{\perp}$ in (9),(10) provide orthogonal projection and preserve the properties $P^2 = P$, $P^{\perp 2} = P^{\perp}$, and $P + P^{\perp} = I$. They are also expressed by

$$P = \begin{bmatrix} I & 0 \\ Y(-L) & 0 \end{bmatrix}, \quad P^{\perp} = \begin{bmatrix} 0 & 0 \\ -Y(-L) & I \end{bmatrix} \quad (53)$$

where $Y(t)$ is the solution to the differential Riccati equation.

$$\begin{aligned}&-\dot{Y}(t) = Y(t)(A - B_2R^{-1}D_{12}^{\mathrm{T}}C)\\&\quad + (A - B_2R^{-1}D_{12}^{\mathrm{T}}C)^{\mathrm{T}}Y(t) - Y(t)B_2R^{-1}B_2^{\mathrm{T}}Y(t)\\&\quad + C^{\mathrm{T}}(I - D_{12}R^{-1}D_{12}^{\mathrm{T}})C, \;\; Y(0) = 0 \end{aligned} \tag{54}$$

■

By Theorem 1, it is shown that the preview compensation law which minimizes (2) is given by (7). Let $e_i$ ($i = 1, 2, \ldots, \tilde{m}_1$) be unit vectors in $\mathbb{R}^{\tilde{m}_1}$, the optimal $H^2$ performance $\gamma_{\rm opt} \geq 0$ is given as follows.

$$\gamma_{\rm opt}^2 = \sum_{i=0}^{\tilde{m}_1} J^{\rm opt}(e_i) = \text{trace}\{\tilde{B}_1^{\rm eT} X \tilde{B}_1^{\rm e}\} \tag{55}$$

## III. Preview Compensation with Partial State Setting

Employing the result stated in Section II, we discuss a design method of the optimal preview compensation law, which consists of the continuous-time input and a sequence of the partial state setting. Define a generalized plant by

$$\begin{aligned}\Sigma_s \quad \dot{x}(t) &= Ax(t) + \sum_{i=0}^{d} B_{1i}w_i(t - h_i) + B_2u(t) \quad (56)\\ z(t) &= C_1x(t) + D_{12}u(t)\\ y(t) &= w(t), \quad x(0) = 0\\ w_i(t) &:= \delta(t)\cdot w_{i0} \;\; (i = 0, 1, \ldots, d)\\ w(t) &:= \begin{bmatrix} w_0^{\mathrm{T}}(t) & w_1^{\mathrm{T}}(t) & \cdots & w_d^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}\\ w_0 &:= \begin{bmatrix} w_{00}^{\mathrm{T}} & w_{01}^{\mathrm{T}} & \cdots & w_{0d}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\\ x(h_i+) &:= x(h_i) + V_iv_{0i} \;\; (i = 0, 1, \ldots, d)\\ v_0 &:= \begin{bmatrix} v_{00}^{\mathrm{T}} & v_{01}^{\mathrm{T}} & \cdots & v_{0d}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\end{aligned}$$

where the command of partial state setting is applied at $t = h_i$ in the case $V_i \neq 0$ ($i = 0, 1, \ldots, d$). In this formulation, it is noted that $\{h_i\}$ denotes the series of time instants such that the disturbance $\delta(t)\cdot w_{0i}$ or the commands of partial state setting $V_iv_{0i}$ are applied . The optimal compensation strategy $(u, v_0)$ which minimizes

$$J_s(u, v_0, w_0) = \|z\|^2_{L_2(0,\infty;\,\mathbb{R}^p)} \tag{57}$$

is obtained by employing Theorem 1.

*Theorem 4:* Suppose (A1),(A2) hold for the system $\Sigma_s$. The optimal compensation strategy $(u^{\rm opt}, v_0^{\rm opt})$ which minimizes (57) is given as follows:

$$u^{\rm opt}(t) = \sum_{k=0}^{d} K(t, h_k)\begin{bmatrix} B_{1k} V_k\end{bmatrix}\begin{bmatrix} w_{0k}\\ v_{0k}^{\rm opt}\end{bmatrix} \tag{58}$$

$$\begin{aligned} v_0^{\rm opt} &= \begin{bmatrix} v_{00}^{\rm optT} & v_{01}^{\rm optT} & \cdots & v_{0d}^{\rm optT}\end{bmatrix}^{\mathrm{T}}\\ &= -(V^{\rm eT}XV^{\rm e})^+(V^{\rm eT}XB_1^{\rm e})w_0 \end{aligned} \tag{59}$$

$$V^{\rm e} = \text{block diag}(V_0, V_1, \ldots, V_d) \tag{60}$$

$$B_1^{\rm e} = \text{block diag}(B_{10}, B_{11}, \ldots, B_{1d}) \tag{61}$$

where $(\cdot)^+$ denotes pseudo-inverse of the matrix and $K(\cdot,\cdot)$ is defined by (8). Furthermore the optimal cost of (57) is expressed as follows.

$$J_s^{\rm opt}(w_0) := J_s(u^{\rm opt}, v_0^{\rm opt}, w_0) = w_0^{\mathrm{T}}B_1^{\rm eT}X_sB_1^{\rm e}w_0 \tag{62}$$

$$X_s := X\{I - V^{\rm e}(V^{\rm eT}XV^{\rm e})^+V^{\rm eT}X\} \tag{63}$$

■

*Proof:* The system $\Sigma_s$ is transformed to $\Sigma$ by rewriting

$$\tilde{B}_{1i} = \begin{bmatrix} V_i & B_{1i}\end{bmatrix}, \;\; \tilde{w}_{0i} = \begin{bmatrix} v_{0i}^{\mathrm{T}} & w_{0i}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \;\; (i = 0, 1, \ldots, d).$$

Then employing Theorem 1, the cost-functional (57) is further optimized in terms of $(u, v_0)$.

$$\begin{aligned} J_s(u, v_0, w_0) &= J_s(u^{\rm opt}, v_0^{\rm opt}, w_0)\\ &\quad + \langle (u - u^{\rm opt}), \Delta(u - u^{\rm opt})\rangle_{L_2(0,L;\,\mathbb{R}^{m_2})}\\ &\quad + (v_0 - v_0^{\rm opt})^{\mathrm{T}}(V^{\rm eT}XV^{\rm e})(v_0 - v_0^{\rm opt}) \quad (64)\\ u^{\rm opt} &= -\Delta^{-1}\begin{bmatrix} \mathcal{G}_0\\ \mathcal{G}_1 + D_{12}\cdot\mathcal{I}\end{bmatrix}^*\begin{bmatrix}\mathcal{F}_0\\ \mathcal{F}_1\end{bmatrix}\begin{bmatrix} v_0^{\rm opt}\\ w_0\end{bmatrix} \quad (65)\\ v_0^{\rm opt} &= -(V^{\rm eT}XV^{\rm e})^+V^{\rm eT}XB_1^{\rm e}w_0 \quad (66)\end{aligned}$$

By (64)-(66) and Theorem 1, the optimal compensation strategy $(u^{\rm opt}, v_0^{\rm opt})$ and the resulting cost are given by (58)-(63). ■

## IV. Typical Preview Control Problems

By Theorem 4, a generalized result of preview compensation law is obtained for a class of stabilized systems. The results stated in Section III cover essential part and enable us to derive a design method of several preview control laws. In this section, we clarify the design methods of (A) $H^2$ full-information preview control law, which includes partial state setting, and (B) the generalized preview compensation law in the servo-mechanism.

### *A. $H^2$ preview full-information control with partial state setting*

Define an $H^2$ preview full-information (FI) control problem by the generalized plant:

$$\begin{aligned}\Sigma_p \quad \dot{x}(t) &= A_px(t) + \sum_{i=0}^{d} B_{1i}w_i(t - h_i) + B_2u(t) \quad (67)\\ z(t) &= C_{p1}x(t) + D_{12}u(t)\\ y(t) &= \begin{bmatrix} x(t)\\ w(t)\end{bmatrix}, \quad x(0) = 0\\ w_i(t) &:= \delta(t)\cdot w_{i0} \;\; (i = 0, 1, \ldots, d)\\ w(t) &:= \begin{bmatrix} w_0^{\mathrm{T}}(t) & w_1^{\mathrm{T}}(t) & \cdots & w_d^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}\\ w_0 &:= \begin{bmatrix} w_{00}^{\mathrm{T}} & w_{01}^{\mathrm{T}} & \cdots & w_{0d}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\\ x(h_i+) &:= x(h_i) + V_iv_{0i} \;\; (i = 0, 1, \ldots, d)\\ v_0 &:= \begin{bmatrix} v_{00}^{\mathrm{T}} & v_{01}^{\mathrm{T}} & \cdots & v_{0d}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\end{aligned}$$

which includes the command of partial state setting. Under the assumptions:

(H1) $(C_{p1}, A_p, B_2)$ is detectable and stabilizable,

(H2) $D_{12}$ has full column rank,

(H3) $\begin{bmatrix} A_p - j\omega I & B_2 \\ C_{p1} & D_{12} \end{bmatrix}$ has full column rank for all $\omega$,

an $H^2$ preview control strategy $(u^{\rm opt}, v_0^{\rm opt})$ which internally stabilizes $\Sigma_p$ and minimizes

$$J_p(u, v_0, w_0) = \|z\|^2_{L_2(0,\infty;\,\mathbb{R}^p)} \tag{68}$$

is given by the following theorem.

*Theorem 5:* Suppose (H1)-(H3) hold for $\Sigma_p$ and let $M \geq 0$ be the stabilizing solution to the Riccati equation:

$$M(A_p - B_2R^{-1}D_{12}^{\rm T}C_{p1}) + (A_p - B_2R^{-1}D_{12}^{\rm T}C_{p1})^{\rm T}M \\ -MB_2R^{-1}B_2^{\rm T}M + C_{p1}^{\rm T}(I - D_{12}R^{-1}D_{12}^{\rm T})C_{p1} = 0. \tag{69}$$

The optimal control strategy $(u^{\rm opt}, v_0^{\rm opt})$ for $\Sigma_p$ and the resulting cost of (68) is expressed as follows:

$$u^{\rm opt}(t) = -R^{-1}(D_{12}^{\rm T}C_{p1} + B_2^{\rm T}M)x(t) \\ - R^{-1}B_2^{\rm T}\sum_{i=j}^{d} e^{A_c^{\rm T}(h_i - t)}M\begin{bmatrix} B_{1k}V_k \end{bmatrix}\begin{bmatrix} w_{0k} \\ v_{0k}^{\rm opt} \end{bmatrix} \\ h_{j-1} \leq t \leq h_j \quad (j = 1, 2, \ldots, d) \tag{70}$$

$$v_0^{\rm opt} = -(V^{\rm eT}XV^{\rm e})^{+}(V^{\rm eT}XB_1^{\rm e})w_0 \tag{71}$$

$$A_c := A_p - B_2R^{-1}(D_{12}^{\rm T}C_{p1} + B_2^{\rm T}M) \tag{72}$$

$$J_p^{\rm opt}(w_0) := J_p(u^{\rm opt}, v_0^{\rm opt}, w_0) = w_0^{\rm T}B_1^{\rm eT}X_sB_1^{\rm e}w_0 \tag{73}$$

where the notations of $R$, $B_1^{\rm e}$, $X_s$ are defined by (12),(61), (63). ■

*Proof:* After the time $t = L$, the system dynamics is reduced to

$$\dot{x}(t) = A_px(t) + B_2u(t) \\ z(t) = C_{p1}x(t) + D_{12}u(t) \tag{74}$$

and, for any driven state $x(L+)$, the optimal control for $t > L$ is given by an LQ control law (e.g.[2]).

$$u^{\rm LQ}(t) = -R^{-1}(D_{12}^{\rm T}C_{p1} + B_2^{\rm T}M)x(t) \tag{75}$$

Rewriting the control input by

$$u(t) = u^{\rm LQ}(t) + \tilde{u}(t), \quad \tilde{u} \in L_2(0, L;\, \mathbb{R}^{m_2}), \tag{76}$$

we will derive an optimal compensation strategy $(\tilde{u}^{\rm opt}, v_0^{\rm opt})$ for the transformed system $\tilde{\Sigma}_p$.

$$\tilde{\Sigma}_p \quad \dot{x}(t) = A_cx(t) + \sum_{i=0}^{d} B_{1i}w_i(t - h_i) + B_2u(t) \tag{77}$$

$$z(t) = C_{c1}x(t) + D_{12}u(t)$$

$$y(t) = \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}, \quad x(0) = 0$$

$$C_{c1} := (I - D_{12}R^{-1}D_{12}^{\rm T})C_{p1} - D_{12}R^{-1}B_2^{\rm T}M$$

$$w_i(t) := \delta(t) \cdot w_{i0} \quad (i = 0, 1, \ldots, d)$$

$$w(t) := \begin{bmatrix} w_0^{\rm T}(t) & w_1^{\rm T}(t) & \cdots & w_d^{\rm T}(t) \end{bmatrix}^{\rm T}$$

$$w_0 := \begin{bmatrix} w_{00}^{\rm T} & w_{01}^{\rm T} & \cdots & w_{0d}^{\rm T} \end{bmatrix}^{\rm T}$$

$$x(h_i+) := x(h_i) + V_iv_{0i} \quad (i = 0, 1, \ldots, d)$$

$$v_0 := \begin{bmatrix} v_{00}^{\rm T} & v_{01}^{\rm T} & \cdots & v_{0d}^{\rm T} \end{bmatrix}^{\rm T}$$

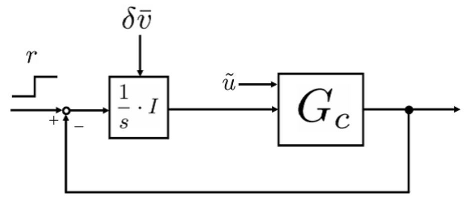


Fig. 1. Servo-mechanism

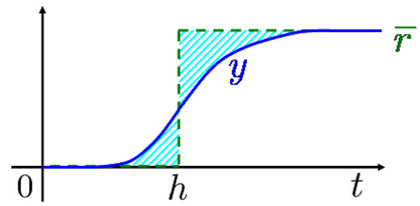


Fig. 2. Step-response

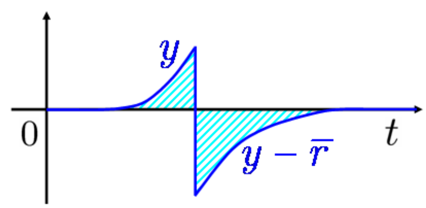


Fig. 3. Regulated response in the preview action

Since the solution $M \geq 0$ to (69) satisfies the Lyapunov equation

$$MA_c + A_c^{\rm T}M + C_{c1}^{\rm T}C_{c1} = 0, \tag{78}$$

the equalities

$$P = \begin{bmatrix} I & 0 \\ M & 0 \end{bmatrix}, \quad P^{\perp} = \begin{bmatrix} 0 & 0 \\ -M & I \end{bmatrix}, \tag{79}$$

$$K(\xi, \eta) = \begin{cases} 0 & \xi \geq \eta \\ R^{-1}B_2^{\rm T}e^{A_c^{\rm T}(\eta - \xi)}M & \xi < \eta \end{cases} \tag{80}$$

are obtained from

$$\begin{bmatrix} -M & I \end{bmatrix} H = -A_c^{\rm T}\begin{bmatrix} -M & I \end{bmatrix}. \tag{81}$$

Thus the optimal control strategy and the resulting cost are given by (70)-(73). ■

In the preview FI control problem $\Sigma_p$, it is observed that the representation of control law is significantly simplified as the kernel of $K(\xi, \eta)$ is reduced to (80). In like manner of Theorem 1, the optimal $H^2$ performance $\gamma_{\rm opt} \geq 0$ is expressed as follows.

$$\gamma_{\rm opt}^2 = \sum_{i=0}^{m_1} J_s^{\rm opt}(e_i) = {\rm trace}\{B_1^{\rm eT}X_sB_1^{\rm e}\} \tag{82}$$

### *B. Preview compensation in the servo-mechanism*

Focus on the servo-mechanism depicted by Fig. 1 which is driven by step reference (Fig. 2). In the case the preview compensation law $(\tilde{u}, \bar{v})$ is applied to the stabilized servo-system, it is expected that the transient is further improved by driving the internal state appropriately. Here, we will

Table 1: Plant parameters

| Sign | Value | Unit | Description |
|---|---|---|---|
| $J_1$ | 0.0109 | [kg·m$^2$] | Moment of Disk 1 |
| $J_2$ | 0.0104 | [kg·m$^2$] | Moment of Disk 2 |
| $c_1$ | 0.006 | [Nms/rad] | Viscous friction of Disk 1 |
| $c_2$ | 0.0006 | [Nms/rad] | Viscous friction of Disk 2 |
| $k$ | 1.4 | [Nm/rad] | Elastic coefficient of torsion spring |
| $G$ | 0.328 | [Nm/V] | Hardware Gain |

show that the generalized compensation law for the servo-mechanism is obtained by Theorem 4.

Describe the servo-mechanism (Fig. 1) by

$$\begin{bmatrix}\dot{x}_c(t)\\ \dot{x}_i(t)\end{bmatrix} = \begin{bmatrix}A_c & B_c\\ -C_c & 0\end{bmatrix}\begin{bmatrix}x_c(t)\\ x_i(t)\end{bmatrix} + \begin{bmatrix}B_c\\ 0\end{bmatrix}\tilde{u}(t) + \begin{bmatrix}0\\ I\end{bmatrix} r(t) \tag{83}$$

$$r(t) := \begin{cases} 0 & 0 \le t \le h \\ \bar{r} & h \le t \end{cases} \tag{84}$$

$$x_i(l+) = x_i(l) + \bar{v}, \quad \tilde{u} \in L_2(0, L; \mathbb{R}^l) \tag{85}$$
$$0 \le h \le L, \ 0 \le l \le L$$

where $r(t)$ is the step reference applied at $t = h$ and $x_i(t)$ is the internal state of the integrator which will be modified by (85). Under the assumption such that the matrix

$$\begin{bmatrix}A_c & B_c\\ -C_c & 0\end{bmatrix} \tag{86}$$

is invertible, we derive the optimal value of the state setting $\bar{v}$ with the optimal compensation input $\tilde{u}$.

Rewriting the state by

$$x(t) = \begin{cases} \begin{bmatrix}x_c(t)\\ x_i(t)\end{bmatrix} & 0 \le t \le h \\ \begin{bmatrix}x_c(t) - x_c(\infty)\\ x_i(t) - x_i(\infty)\end{bmatrix} & h < t \end{cases}, \tag{87}$$

where $x_c(\infty)$, $x_i(\infty)$ are the equilibrium states defined by

$$\begin{bmatrix}x_c(\infty)\\ x_i(\infty)\end{bmatrix} = -\begin{bmatrix}A_c & B_c\\ -C_c & 0\end{bmatrix}^{-1}\begin{bmatrix}0\\ I\end{bmatrix}\bar{r}, \tag{88}$$

the optimization of $(\tilde{u}, \bar{v})$ is formulated by the system

$$\tilde{\Sigma}_s: \begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t-h) + B_2\tilde{u}(t) \\ z(t) &= C_1 x(t) + D_{12}\tilde{u}(t) \\ y(t) &= w(t), \quad x(0) = 0 \\ x(l+) &= x(l) + E\bar{v} \\ w(t) &= \delta(t)\cdot\bar{r} \\ A &= \begin{bmatrix}A_c & B_c\\ -C_c & 0\end{bmatrix}, B_1 = A^{-1}\begin{bmatrix}0\\ I\end{bmatrix} \\ B_2 &= \begin{bmatrix}\tilde{B}_c\\ 0\end{bmatrix}, V = \begin{bmatrix}0\\ I\end{bmatrix}\end{aligned} \tag{89}$$

with the cost-functional $\tilde{J}_s(u, \bar{v}, \bar{r}) = \|z\|^2_{L_2(0,\infty)}$. The optimal strategy of $(u, \bar{v})$ is obtained by Theorem 4 and it is directly applied to (83) as the internal state of $\tilde{\Sigma}_s$ is not needed.

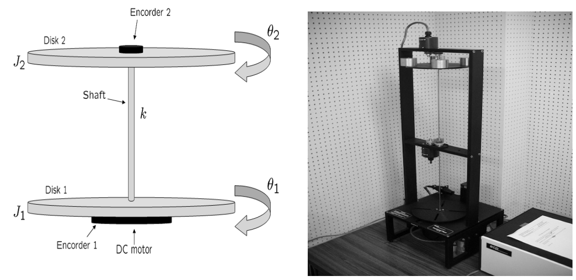


Fig. 4. Two-inertia system

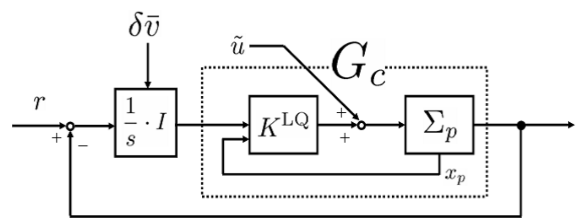


Fig. 5. Servo-mechanism

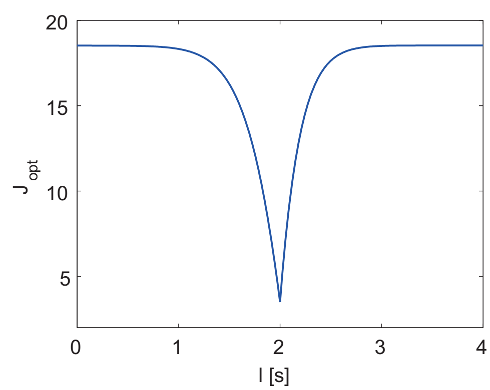


Fig. 6. Relation between the state setting time and $J_{\mathrm{opt}}$

## V. Design Example

For the two-inertia system depicted by Fig. 4, we will design a servo-mechanism and investigate the feature of the compensation law, which includes the command of partial state setting. The control objective is to attain favorable transient motion of Disk 2 by applying the torque to Disk 1. Both disks are connected by torsion spring and it is expected that the appropriate preview motion improves the transient of the whole system.

The two-inertia system is described by the state equation:

$$\Sigma_p : \begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u(t) \\ y(t) &= C_p x_p(t) \end{aligned} \tag{90}$$

$$A_p := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{J_1} & \frac{k}{J_1} & -\frac{c_1}{J_1} & 0 \\ \frac{k}{J_2} & -\frac{k}{J_2} & 0 & -\frac{c_2}{J_2} \end{bmatrix}, \ B_p := \begin{bmatrix} 0 \\ 0 \\ \frac{G}{J_1} \\ 0 \end{bmatrix}$$

$$C_p := \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \ x_p(t) := \begin{bmatrix}\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\end{bmatrix}^{\mathrm{T}}$$

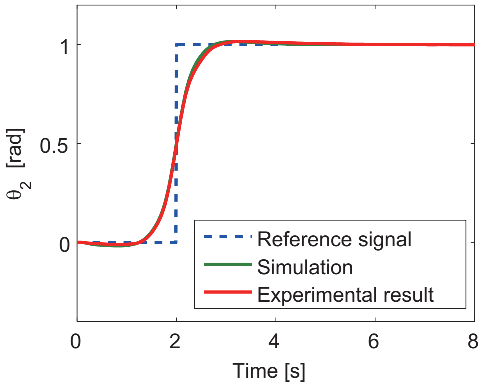


Fig. 7. Response (preview compensation with state-setting)

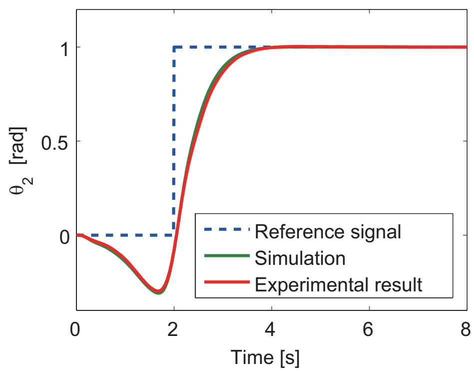


Fig. 8. Response (preview compensation)

where the parameters are summarized by Table 1. Further including the internal state of the integrator $x_i(t)$ (Fig. 5), the augmented system is formulated in the following form.

$$\dot{x}_s(t) = \begin{bmatrix} A_p & 0 \\ -C_p & 0 \end{bmatrix} x_s(t) + \begin{bmatrix} B_p \\ 0 \end{bmatrix} u(t) \qquad (91)$$

$$x_s(t) = \begin{bmatrix} x_p(t) \\ x_i(t) \end{bmatrix}$$

For (91), we introduce an LQ control law

$$u(t) = K^{\mathrm{LQ}} \begin{bmatrix} x_p(t) \\ x_i(t) \end{bmatrix} + \tilde{u}(t) = \begin{bmatrix} K_p^{\mathrm{LQ}} & K_i^{\mathrm{LQ}} \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_i(t) \end{bmatrix} + \tilde{u}(t) \qquad (92)$$

which minimizes

$$J = \int_0^\infty \{x_s^{\mathrm{T}}(t) Q x_s(t) + u^{\mathrm{T}}(t) R u(t)\}\, dt \qquad (93)$$

$$Q = \mathrm{diag}(10, 10, 1, 1, 100), R = 1$$

and investigate the effect of preview compensation $\tilde{u}$ and the state setting of the integrator $\bar{v}$. In the simulation and the experimental results, the step reference (84) is applied at $t = 2.0$[s] ($\bar{r} = 1, h = 2$).

By (91), (92) with the regulated output:

$$z(t) = \begin{bmatrix} Q^{\frac{1}{2}} \\ R^{\frac{1}{2}} K^{\mathrm{LQ}} \end{bmatrix} x_s(t) + \begin{bmatrix} 0 \\ R^{\frac{1}{2}} \end{bmatrix} \tilde{u}(t), \qquad (94)$$

the preview compensation strategy $(\tilde{u}, \bar{v})$ is obtained along Section IV-B.

By Theorem 4, the relation between the time of state setting $(t = l)$ and the optimal performance $J_{\mathrm{opt}}$ is summarized by Fig. 6. In this example, it is observed that the state setting works most effectively when it is applied simultaneously with the step reference.

Fig. 7 shows the responses when the compensation $(\tilde{u}, \bar{v})$ is applied and the transient is fairly improved by attenuating over-and-under shoot ($J_{\mathrm{opt}} = 3.50$). While Fig. 8 shows the responses when the state of the integrator is not modified (the compensation $\tilde{u}$ is solely optimized by Theorem 1). This case, the undershoot is not well attenuated and there remains a room for further improving the transient ($J_{\mathrm{opt}} = 18.52$).

## VI. Conclusion

A generalized $H^2$ preview control problem, which includes the commands of partial state setting, is discussed and the optimal $H^2$ preview control strategy and the achievable performance are clarified based on the state-space characterization. The approach adopted here is also applicable for the improvement of $H^2$ control for multiple delay systems.

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