Networked Adaptive Model Predictive Control

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Abstract— **This paper presents a neighborhood based networked predictive adaptive controller that is suitable for control of multivariable systems made of subsystems that interact sequentially and on its demonstration using simulations in a detailed nonlinear model of a water delivery canal. Furthermore some convergence properties of the adaptive algorithm are proved.**

According to the approach followed, the overall system is decomposed in local systems. To each local system, a local adaptive predictive controller that manipulates its input is associated. In order to improve the overall performance, communication between local controllers is provided through feedforward terms from adjacent local systems.

I. INTRODUCTION

Networked MPC currently attracts an increasing attention. This is due to well known properties underlying MPC, such as its capability to tackle constraints, non-minimum phase effects and uncertainty in plant transport delay [1], [2], [5], [6], [10], [13]. The inclusion of adaptive features has however not yet been considered in a distributed framework and is the main subject of this work.

Water distribution networks based on open canals, used *e. g.* for agricultural purposes, are large plants that, due to their distributed character, present complex dynamics. They are formed by a number of interconnected water pools, limited by gates. In the cases considered hereafter, the water level along the canal reaches is modeled by the Saint-Venant equations, a pair of partial differential equations that embed mass and momentum conservation, together with boundary conditions that may be manipulated to some extent by the canal gate positions. Due to their large space dimensions they form a natural field of application of networked control and distributed optimization. This may be motivated either by the need to make a distributed implementation or by exogenous factors, such as the canal crossing different administrative districts, each one requiring their own command rules, but needing coordination.

As such, among the rich bibliography on canal control, it is possible to find many examples of distributed algorithms, including distributed MPC, optimal LQ with augmented lagrangian and several forms of simplification of multivariable control to reduce the coupling among decision centers [3], [4], [7], [11], [12].

Methods based on neighborhood optimization [13] and control [7] are specially interesting for water delivery canals. In these structures, a local controller is associated to each pool and adjacent units are assumed to interchange information.

The contribution of this paper consists in a neighborhood based networked predictive adaptive controller that is suitable for control of water delivery networks of open canals and on its demonstration using simulations in a nonlinear model of a pilot canal. Algorithm properties are proved as well.

The networked controller proposed may be seen as a reduced complexity controller, where a number of gains are forced to vanish. It is shown that, if the prediction horizon is large enough, each of the local adaptive controllers with feedforward terms adjust the gains in a ”Newton direction” in the process of minimizing the underlying quadratic cost given the information pattern available in terms of sensor measurements.

The paper is organized as follows: After this Introduction that motivates the problem, briefly reviews the literature and presents the paper contributions and structure, section 2 presents the proposed approach to adaptive predictive networked control. Some convergence properties if this control structure are then stated in section 3 and proved in the Appendix. Section 4 describes the canal considered and provides simulation results for controller evaluation. Finally, section 5 draws conclusions.

II. NETWORKED CONTROL

A water delivery canal is a large scale distributed system, in which the different subsystems (water pools, separated by gates) interact sequentially. In the networked control structure proposed a local controller is associated to each pool (fig. 1). In addition to the feedback interconnection with the corresponding pool, local controller C_i, $i = 2, \dots, 4$ receives information from C_{i-1}. Two possibilities considered hereafter consist in the tracking error or the tracking error sum of previous pools. This is used as a feedforward action that advances disturbance actions induced by the local controllers of previous pools when correcting the level.

To each pool i, $i = 1, \dots 4$ associate a receding horizon cost defined by

$$J_i(t) = E\left[\sum_{j=1}^{T} \tilde{y}_i^2(t+i) + \rho \sum_{j=1}^{T-1} u_i^2(t+j-1)|I^t\right] \quad (1)$$

where the tracking error $\tilde{y}_i$, with respect to the desired set-point $y^*(t)_i$ is given by

$$\tilde{y}_i(t) = y_i^*(t) - y_i(t) \quad (2)$$

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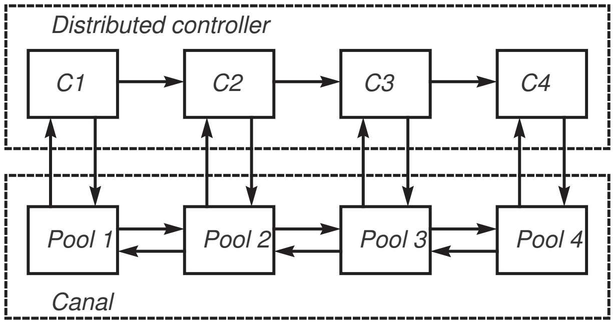

Fig. 1. Distributed control of a water delivery canal.

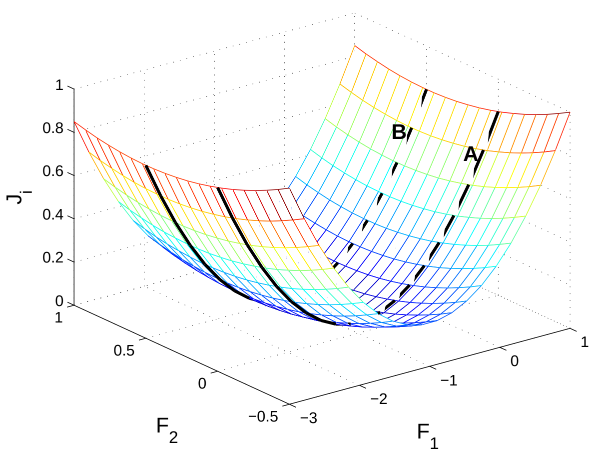

Fig. 2. Explanation of the performance increase due to neighbor controller information.

This aims at yielding a controller that, for T large enough, approximates the minimum of the steady-state cost

$$J_{\infty,i} = \lim_{t\to\infty} E[\tilde{y}_i^2(t) + \rho u_i^2(t)] \tag{3}$$

The networked controller may be seen as a reduced complexity controller, where a number of gains are forced to vanish. Fig. 2 helps explaining this concept in a simple situation. Suppose that, for a single pool, only one feedback gain (F_1) is used. As shown in the schema of fig. 2, the cost as a function of the gain corresponds to the curve labeled A. If the controller is allowed an extra degree of freedom by the inclusion of the feedforward gain F_2, the cost as a function of the original gain F_1 (curve B) will present a minimum that is smaller than the one of curve A. Actually, curves A and B are part of the cost function map J_i for pool i, constrained to the chosen controller structure.

A key issue is whether the adaptation mechanism is able to drive the gains to the minimum of the underlying cost, constrained to the chosen structure, *i. e.* imposing that only information from the previous local controller is used, in addition to local feedback measures. Although it is not possible that the following algorithm converges, it is shown in section 4 that it evolves in a Newton direction with respect to the constrained cost surface.

A. Modified MUSMAR algorithm

For each pool, the modified MUSMAR algorithm reads as follows (in order to alleviate the notation, the pool index is omitted):

At the beginning of each sampling interval t (discrete time), recursively perform the following steps:

1.Sample plant output, $y(t)$ and use 2 to compute the tracking error $\tilde{y}$, with respect to the desired set-point $y^*(t)$.

2.Using Recursive Least Squares (RLS), update the estimates of the parameters θ_j, ψ_j, μ_{j-1} and ϕ_{j-1} in the following set of predictive models:

$$\tilde{y}(t+j) \approx \theta_j u(t) + \psi_j' s(t) \tag{4}$$

$$u(t+j-1) \approx \mu_{j-1} u(t) + \phi_{j-1}' s(t) \tag{5}$$

$$j = 1, \dots, T$$

where $\approx$ denotes equality in least squares sense and $s(t)$ is a sufficient statistic for computing the control, hereafter referred as the pseudo-state, given by

$$s(t) = [\tilde{y}(t)\ \dots\ \tilde{y}(t-n+1)\ u(t-1)\ \dots\ u(t-m)$$

$$w(t)\ \dots\ w(t-n_{w1})]' \tag{6}$$

where the w are samples of the feedforward signal incoming from the previous pool. Since, at time t, $\tilde{y}(t+j)$ and $u(t+j)$ are not available for $j \geq 1$, for the purpose of estimating the parameters, the variables in (4,5) are delayed in block of T samples. The estimation equations are thus,

$$K(t) = \frac{P(t-1)\varphi(t-T)}{1+\varphi'(t-T)P(t-1)\varphi(t-T)[1-\beta(t)]} \tag{7}$$

$$P(t) = [I - K(t)\varphi'(t-T)(1-\beta(t))]P(t-1) \tag{8}$$

where, for $j = 1, \dots, T$:

$$\hat{\Theta}_j(t) = \hat{\Theta}_j(t-1) +$$

$$+K(t)[y(t-T+j) - \hat{\Theta}_j(t-T)'\varphi(t-T)] \tag{9}$$

and for $j = 1, \dots, T-1$:

$$\hat{\Omega}_j(t) = \hat{\Omega}_j(t-1) +$$

$$+K(t)[u(t-T+j) - \hat{\Omega}_j(t-T)'\varphi(t-T)] \tag{10}$$

In these equations, $\hat{\Theta}_j$ represents the estimate of the parameter vector of the output predictors, given at each discrete time and for each predictor j by

$$\hat{\Theta}_j = [\theta_j\ \psi_j']'$$

and $\varphi(t-T)$ represents the regressor, common to all predictors, given by

$$\varphi(t-T) = [u(t-T)\ s'(t-T)]'$$

Similarly, $\hat{\Omega}_j$ represents the estimate of the parameter vector of the input predictors, given at each discrete time and for each predictor j by

$$\hat{\Omega}_j = [\mu_j\ \phi_j']'$$

Note that, since the regressor $\varphi(t-T)$ is common to all the predictive models, the Kalman gain update (7) and the covariance matrix update (8) are also common to all the predictors and need to be performed only once per time iteration. This greatly reduces the computational load.

The variable $\beta(t)$ denotes the quantity of information discarded in each iteration, and is computed according to a directional forgetting scheme by

$$\beta(t) = 1 - \lambda + \frac{1-\lambda}{\varphi'(t-T)P(t-1)\varphi(t-T)}$$

where λ is a constant to be chosen between 0 (complete forgetting) and 1 (no forgetting) which determines the rate of forgetting in the direction of incoming information. In practice, a factorized version is used to implement eq. (7).

3.Apply to the plant the control given by

$$u(t) = F's(t) + \eta(t) \tag{11}$$

where η is a white dither noise of small amplitude and F is the vector of controller gains, computed from the estimates of the predictive models by

$$F = -\frac{1}{\alpha}(\sum_{j=1}^{T}\theta_j\psi_j + \rho\sum_{j=1}^{T-1}\mu_j\phi_j) \tag{12}$$

with the normalization factor α given by

$$\alpha = \sum_{j=1}^{T}\theta_j^2 + \rho(1 + \sum_{j=1}^{T-1}\mu_j^2) \tag{13}$$

B. Guidelines for MUSMAR configuration.

The previous algorithm is a modification of MUSMAR [9] that incorporates feedforward terms. The MUSMAR controller is based on a number of separately estimated predictive models. In the presence of plant/model mismatches, such as the situations found here, the redundancy thereby introduced proves important for achieving a correct control action. MUSMAR is equivalent to a bank of parallel self-tuners, each one tuned to a different value of plant delay and with different weights. If the actual plant delay is bigger than the delay assumed for a given self-tuning channel the corresponding weight will be zero. Insensitivity to uncertainty in plant delay is thus achieved up to some degree.

The choice of the variables and the number of their past samples entering $s(t)$ defines the structure of the controller. The choice of n and m should be such that it allows to capture the dominant dynamics of the system. Too big values of n and m imply more parameters to estimate and this may lead to identifiability problems, in turn causing loss of control performance. The value of the prediction horizon T should be large enough so that the gains are close approximations to steady-state (infinite horizon) LQ optimal gains. However, if T is too large, predictive model parameter estimates loose accuracy and this results in gain de-tuning and consequent loss of performance. A trade-off has thus to be made for choosing T.

III. Convergence properties

The convergence properties of the adaptive algorithm applied to each of the local controllers are summarized in the following propositions whose proofs are shown in the appendix.

Proposition 1. Let F_{k-1} be the controller gains used in the local controller connected to pool i, $i = 1, \ldots 4$ (in order to avoid overloading the notation the index i is omitted). These are supposed to be applied for a number of past iterations long enough such that the estimator has converged and let the vector of update parameters F_k be computed from (12) on such a way that the predictor parameters are replaced by their least-squares estimates at equilibrium. Then,

$$F_k = F_{k-1} - \frac{1}{2\alpha(F_{k-1})}R_s^{-1}(F_{k-1})\nabla_T J_i(F_{k-1}) \tag{14}$$

where

$$\alpha(F_{k-1}) \triangleq \sum_{i=1}^{T}\theta_i^2(F_{k-1}) + \rho\left(1 + \sum_{i=1}^{T-1}\mu_i^2(F_{k-1})\right), \tag{15}$$

$$R_s(F_{k-1}) \triangleq lim_{t\to\infty}E[s(t)s'(t)] \tag{16}$$

and $\nabla_T(F_{k-1})$ is the gradient of the receding horizon cost (1), computed for $F = F_{k-1}$.

□

This proposition states that, if the prediction horizon is large enough, each of the local adaptive controllers with feedforward terms adjust the gains in a "Newton direction" in the process of minimizing the underlying quadratic cost (3).

In general, nothing can be proved about the convergence properties of the MUSMAR algorithm. However, if T is large enough, $\nabla_T J_i(F)$ is a tight approximation to the gradient of the steady state cost (3), $\nabla J_\infty(F)$, and the controller gains are then updated by

$$F_k = F_{k-1} - \frac{1}{2\alpha(F_{k-1})}R_s^{-1}(F_{k-1})\nabla J_\infty(F_{k-1}) \tag{17}$$

In this case, the following proposition holds:

Proposition 2. For the controller associated to a given pool, let the controller gains be updated by (17). Then, the only possible convergence gains are local minima of the steady-state underlying cost (3), constrained to the local controller structure imposed by the choice of the pseudo-state s.

□

IV. Case study: Water delivery canal

A. The canal pilot plant

The canal taken as prototype (figure 3) is a large scale experimental plant that belongs to Núcleo de Hidrulica and Controlo de Canais (Universidade de Évora, Portugal). As shown in figure 4, this canal has four branches (pools) separated by three orifice gates, with the last having an overshoot gate downstream. The design flow is $0.09\,\mathrm{m^3s^{-1}}$. There are water off-takes downstream from each branch that

Fig. 3. A view of the pilot canal.

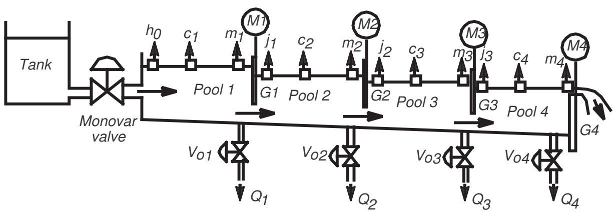

Fig. 4. Schematics of the pilot canal.

are orifices in the channel walls, with additional pipes and valves and equipped with flowmeters.

In order to make possible real time digital control, water level sensors are installed, three for each pool, respectively, at the beginning, middle and end of the pool. Each sensor is installed within an off-line stilling well seen in fig. 3. In this pipe, a curve with $90°$ and of the same diameter was installed, for the connection to the channel (near the bottom). These sensors allow to record variations around $0.7\,mm$ in the water levels. For gate number i, $i = 1, \ldots, 4$, the upstream level is denoted M_i and the downstream level J_i (figure 4).

The canal inlet is equipped with a motorized flow control valve that delivers a specified discharge to the canal input.

B. Controller evaluation

In order to perform the comparative evaluation of the networked control structures considered, in respect to the performance achieved by isolated local controllers, simulations have been made using a nonlinear model of the pilot canal. The model embeds the Saint-Venant equations for the flow in the pools, together with suitable boundary conditions and was validated against plant data.

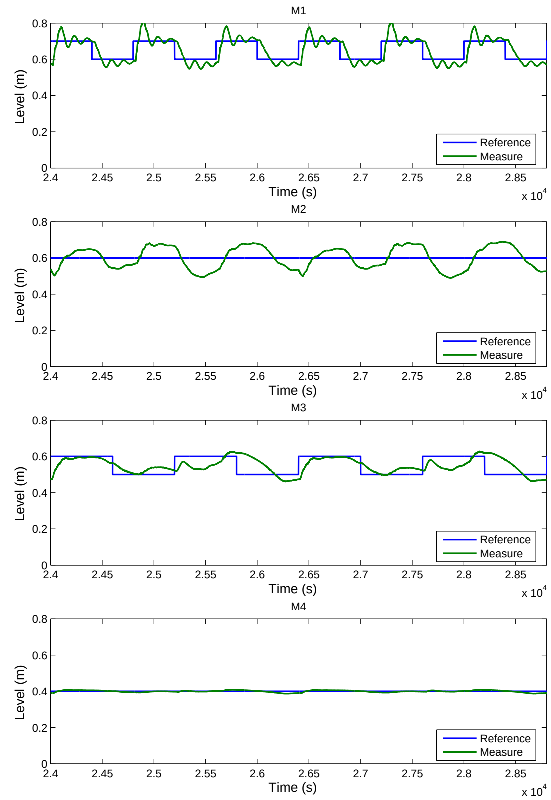

Fig. 5. Set-point tracking by MUSMAR with downstream simple local control.

In what concerns controller configuration parameters, the sampling interval is 5 seconds, $T = 10$, $\rho = 0.01$, $n = 3$, $m = 2$ and $\lambda = 0.995$. These parameters were selected by trial and error in a interval of values that yields the best results.

Both (proximal) upstream and (distant) downstream control have been considered. In each case, the addition of feedforward terms has been included in order to include communication between local controllers. In the case of upstream control two possibilities have been considered for feedforward: Use an accessible disturbance measured by the tracking error of the previous pool level and accessible disturbance measured by the addition of all the previous pool level tracking errors. Figures 5, 6 and 7 illustrate some of the results obtained.

Table 1 shows the results taking as base line reference the loss obtained with isolated upstream controllers. The left column refers to the reference tracking problem and the right column to the problem of rejecting disturbances induced by the lateral off-takes. In the reference tracking problem, the reference is made to vary in pools 1 and 2 and is kept constant, as shown in fig. 5. In the disturbance rejection problem the reference level is kept constant in all pools and the out flow of pools 1 and 3 is made to vary as shown in fig. 7. The out flow of pools 2 and 4 is kept at zero.

From the results in table 1 it is concluded that the use of communication between local controllers always improves the performance. This improvement is of the order of 10%

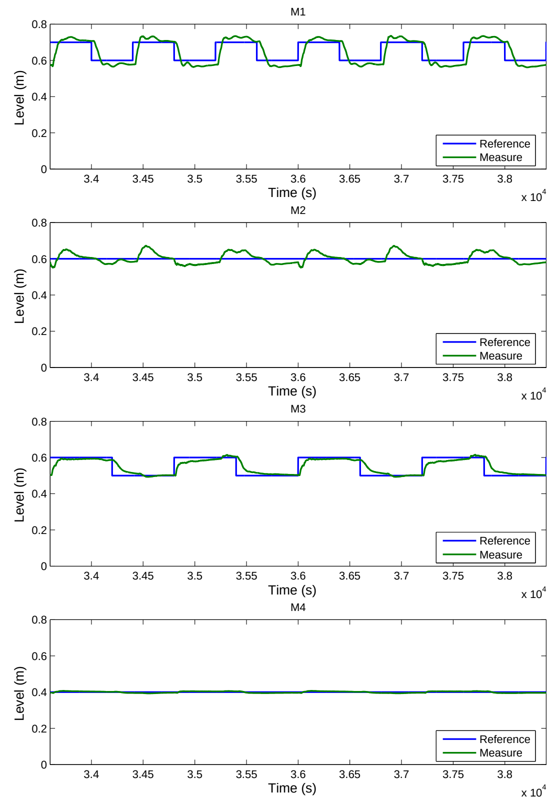

Fig. 6. Set-point tracking by MUSMAR with downstream control and feedforward from accessible disturbances.

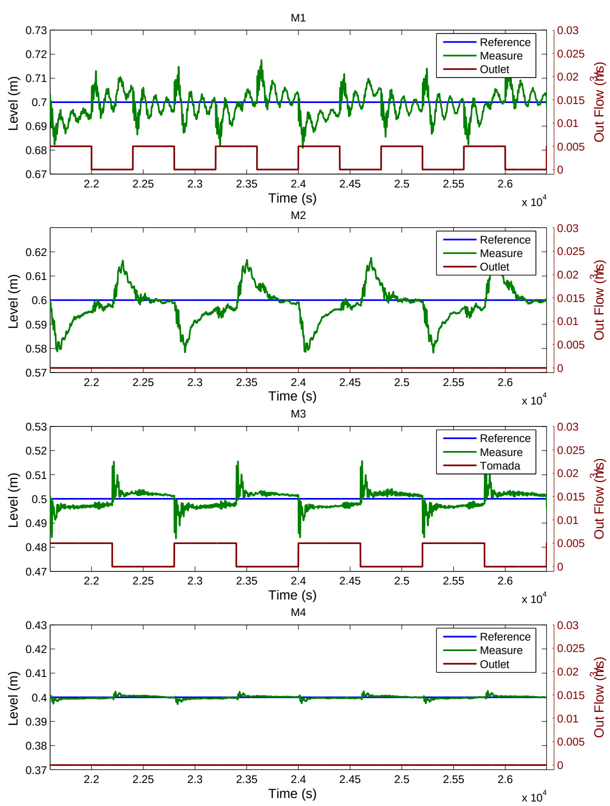

Fig. 7. Disturbance rejection by MUSMAR with downstream control and feedforward from accessible disturbances.

Controller	**Ref. Track**	**Dist. Rej.**
Upstream	100	100
Upstream with accessible dist.	97	91
Upstream with added access. dist.	89	93
Downstream	91	74
Downstream with accessible dist.	81	70

TABLE I

RELATIVE LOSSES OF THE DIFFERENT CONTROL METHODS [%].

in the reference tracking problem, being smaller for the disturbance rejection problem. The improvement is also clear when comparing figures 5 (only local control) and 6 (networked control).

V. CONCLUSION

The use of a networked control structure where local controllers receive information from neighbor controllers yields an increased performance when applied to water delivery canals. In general, this does not guarantee a stable closed-loop system. However, it is shown that, if stability can be achieved with the structure given, the predictive adaptive algorithm described will seek to minimize an underlying quadratic cost along a Newton direction. The only possible equilibria are close approximations to the local minima of the underlying cost, constrained to the imposed controller structure. This type of approach may also be applied to other types of plants.

VI. ACKNOWLEDGEMENT

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APPENDIX

Proof of Proposition 1

According to the control strategy used

$$\tilde{y}(t+j) \approx \theta_j(F_{k-1}) + H'_j(F_{k-1},\, F_k)s(t) \tag{18}$$

$$u(t+j-1) \approx \mu_{j-1}(F_{k-1})\eta(t) + G'_{j-1}(F_{k-1},\, F_k)s(t) \tag{19}$$

where

$$H_j(F_{k-1},\, F_k) = \psi_j(F_{k-1}) + \theta_j(F_{k-1}F_k \tag{20}$$

$$G_{j-1}(F_{k-1},\, F_k) = \phi_{i-1}(F_{k-1})F_k \tag{21}$$

Let F_k be computed according to (12, 13). By adding and subtracting $\theta_j^2(F_{k-1}F_{k-1})$ and $\mu_i^2(F_{k-1})F_{k-1}\rho$, this becomes:

$$F_k = -\frac{1}{\alpha(F_{k-1})}\left(\sum_{j=1}^{T}\theta_j(F_{k-1})H_j(F_{k-1},\, F_{k-1}) + \right.$$
$$\left. +\rho\sum_{j=1}^{T-1}\mu_j(F_{k-1})G_j(F_{k-1},\, F_{k-1})\right) +$$
$$+\frac{F_{k-1}}{\alpha(F_{k-1})}\left(\sum_{j=1}^{T}\theta_j^2(F_{k-1}) + \rho\sum_{j=1}^{T-1}\mu_i^2(F_{k-1})\right) \tag{22}$$

If $\rho F_{k-1}/\alpha(F_{k-1})$ is added and subtracted, (22) becomes

$$F_k = -\frac{1}{\alpha(F_{k-1})}\left(\sum_{j=1}^{T}\theta_j(F_{k-1})H_j(F_{k-1},\, F_{k-1}) + \right.$$
$$\left. +\rho\sum_{j=1}^{T-1}\mu_j(F_{k-1})G_j(F_{k-1},\, F_{k-1}) + \rho F_{k-1}\right) +$$
$$+\frac{F_{k-1}}{\alpha(F_{k-1})}\left(\sum_{j=1}^{T}\theta_j^2(F_{k-1}) + \right.$$
$$\left. +\rho\left(1 + \sum_{j=1}^{T-1}\mu_i^2(F_{k-1})\right)\right) \tag{23}$$

The gradient $\nabla_T J(F)$ of the receding horizon cost $J_T(t)$ with respect to the controller gains is given by

$$\nabla_T J(F) = E\left[\sum_{j=1}^{T}\tilde{y}(t+j)\frac{\partial\tilde{y}(t+j)}{\partial F} + \right.$$
$$\left. +\rho u(t+j-1)\frac{\partial u(t+j-1)}{\partial F}\right]$$

$$= \sum_{j=1}^{T}(\theta_i(F_{k-1})E[y(t+j)s(t)] +$$
$$+\rho\mu_{j-1}(F_{k-1})E[u(t+j-1)s(t)]) \tag{24}$$

or, equivalently, by

$$\frac{1}{2}R_s^{-1}\nabla_T J(F) = \sum_{i=1}^{T}(\theta_i(F_{k-1})H_i(F_{k-1},\, F_k)$$
$$+\rho\mu_{j-1}(F_{k-1})G_{j-1}(F_{k-1},\, F_k)) \tag{25}$$

where ((20, 21, 13) are used together with

$$E[y(t+j)s(t)] = R_s H_j(F_{k-1},\, F_k) \tag{26}$$

$$E[u(t+j)s(t)] = R_s G_j(F_{k-1},\, F_k) \tag{27}$$

Since $\mu_0 = 1$, $\phi_0 = 1$ and $G_0(F_{k-1},\, F_{k-1})$, the conclusion follows from (25) and (23).

q. e. d.

Proof of Proposition 2.

Let iterations (17) be initialized in a neighborhood of a local minimum F^* of J_∞, small enough so that J_∞ is unimodal in it. In this region define the candidate Lyapunov function

$$V(F) = J_\infty(F) - J_\infty(F^*) \tag{28}$$

This function is continuous and differentiable with respect to F and vanishes for $F = F^*$, at which point it has a local minimum. Furthermore, it is strictly decreasing around the trajectories of (17). Indeed:

$$V(F_k) - V(F_{k-1} = J_\infty(F_k) - J_\infty(F_{k-1}) =$$
$$= (\nabla J_\infty(F_{k-1}))' \cdot (F_k - F_{k-1}) + o(F_k - F^*)$$
$$= -\frac{1}{\alpha(F_{k-1})}(\nabla J_\infty(F_{k-1}))' R_s^{-1}(F_{k-1})\nabla J_\infty(F_{k-1}) +$$
$$+o(F_k - F^*) \tag{29}$$

Hence, and since it can be proved that $R_s > 0$ [9], in a sufficiently small neighborhood of F^*,

$$V(F-k) - V(F_{k-1}) < 0 \tag{30}$$

and by Lyapunov's Direct Method [8] the sequence F_k converges to F^*.

In an analogous way, around a local maximum F_{max} one can define the function

$$W(F) = -J_\infty(F) + J_\infty(F_{max}) \tag{31}$$

This function is continuous and differentiable and has a local minimum at F_{max} but now, in a neighborhood of the maximum, it may be proved using similar arguments as above that (17) yields

$$W(F_k) - W(F_{k-1}) > 0 \tag{32}$$

This allows to conclude that F_{max} is unstable in the sense of Lyapunov. Hence, the recursion (17) does not yield a sequence of gains that converge to a maximum.

q. e. d.