Synchronization and pinning control of networks via adaptation and edge snapping

Pietro DeLellis, Mario diBernardo and Maurizio Porfiri

Abstract— **In this paper, we propose novel adaptive pinning control strategies for synchronization of complex networks. The novelty of theses approaches is the adaptive selection of pinned nodes along with the fully decentralized adaptation of the coupling and control gains. The effectiveness of the proposed strategies is validated with numerical simulations on a testbed example.**

I. INTRODUCTION

Network synchronization can be observed in a wide range of contexts including biology, sociology and technology [1], [2], [3], [4]. Synchronization is also a common experience in human life, from metabolic processes to the human interactions. Moreover, synchronization can be found in many man-made devices, such as pendulum clocks, musical instruments, lasers, and electronic power systems (for a description of some representative examples, see [5]).

In synchronization problems, the dynamics of each isolated node in the network can be represented by a nonlinear differential equation of the form $\dot{x}_i = f(x_i, t)$, where $x_i \in \mathbb{R}^n$ is the state of the i^{th} system and $f : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is the vector field describing the node individual dynamics. The topology of the interconnection among individuals can be described by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. Combining the individual dynamics with the topological interactions, the following network model was developed in the late Nineties [1]:

$$\dot{x}_i = f(x_i, t) + \sigma \sum_{j=1}^{N} a_{ij}(h(x_j) - h(x_i)), \qquad i = 1, \ldots, N, \tag{1}$$

where $h : \mathbb{R}^n \to \mathbb{R}^n$ is the output function describing the information exchanged among the network nodes, σ is the coupling strength, and a_{ij} is the ij^{th} element of the adjacency matrix $\mathcal{A}$ associated to graph $\mathcal{G}$.

From a control viewpoint, we typically want all the systems' trajectories to converge onto a desired one, denoted with $x_s(t)$; such problems are typically encountered in formation control [6], [7], [8]. To achieve this goal, the so-called pinning control [9], [10], [11], [12] technique was introduced: an external node (the *pinner*) is added to the network and connected only to a small fraction of the network nodes. The equation describing the closed-loop network dynamics are:

$$\begin{aligned}
\dot{x}_i &= f(x_i, t) + \sigma \sum_{j=1}^{N} a_{ij}(h(x_j) - h(x_i)) + \\
&\quad \delta_i q_i (h(x_j) - h(x_i)), \qquad i \in \mathcal{V} \qquad (2)\\
\dot{x}_s &= f(x_s, t) \qquad (3)\\
\delta_i &= \begin{cases} 1, & i = 1, ..., N_{pin}, \\ 0, & i = (N_{pin} + 1), ..., N. \end{cases} \qquad (4)
\end{aligned}$$

The convergence of all trajectories to the reference established by the pinner requires a proper selection of the coupling gain σ, the control gains q_i and the number of pinned nodes N_{pin}. The selection of the network parameters is often difficult and, moreover, requires a complete information on the network topology, as clearly shown in [13].

In this paper, we aim at overcoming these problems by using adaptive techniques. In particular, the modulation of the coupling and control gains is determined through the decentralized strategy firstly proposed in [14], [15], while the selection of the active gains is performed through the edge snapping technique presented in [16]. The combination of these adaptation strategies drives the network to the desired synchronization trajectory in a fully decentralized way by selecting adaptively the nodes to be controlled and the intensity of the coupling. The effectiveness of the presented approach is validated numerically on a testbed example.

II. PINNING CONTROL VIA EDGE SNAPPING

Controlling a complex network through pinning control requires selecting (i) the number and location of pinning sites and (ii) the numerical values of control gains. Addressing these issues requires a complete knowledge of the node dynamics and the coupling configurations. In what follows, we first assume that the control gains are fixed a priori and we propose the so-called edge snapping technique to adaptively make decisions (i) and later we release this assumption and combine edge snapping with a decentralized adaptation mechanism to address (ii).

A. Fixed control gains

In the classical pinning control scheme, the pinned nodes are selected a priori through (4). Here, inspired by the edge snapping mechanism introduced in [16], we drive the selection of the pinned nodes through the evolution of the network. Namely, δ_i is not anymore a binary constant value, but its evolution is described by the following equation:

$$\ddot{\delta}_i + \zeta \dot{\delta}_i + \frac{d}{d\delta_i} V(\delta_i) = g(e_i), \quad i \in \mathcal{V}. \tag{5}$$

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Here, d is the damping parameter and δ_i is modeled as a unitary mass in a double-well potential V subjected to an external force g, function of the pinning error $e_i = x_s - x_i$.

A possible simple choice for the potential is the following:

$$V(z) = bz(z-1)^2, \tag{6}$$

where b is a parameter defining the height of the barrier between the two wells. With this simple choice of the potential, the dynamical system (5) has only two stable equilibria, that are 0 and 1. These states correspond respectively to pinning or not the corresponding node i.

To show the effectiveness of this approach, we consider a network of 30 Lorenz oscillators [17], [18], [19], coupled through a Erdös and Rényi (ER) random graph [20]. The dynamics of an isolated node is described by the following set of three differential equations:

$$\dot{p} = a_1(q-p) \tag{7}$$
$$\dot{q} = a_2 p - q - pr \tag{8}$$
$$\dot{r} = pq - a_3 r, \tag{9}$$

where $x = [p, q, r]^T$ is the state vector and a_1, a_2 and a_3 are three positive parameters. In our simulations, to ensure the chaotic behavior of the system, we choose $a_1 = 10$, $a_2 = 28$ and $a_3 = 8/3$. Moreover, we assume that the oscillators are diffusively coupled coupling on all the three variables, namely $h(x) = x$. The initial conditions on the network nodes are taken from a uniform distribution between 0.5 and 2.5, while the initial conditions for the pinner are $x_s(0) = [0, 1, 2]^T$. We further assume that the network is weakly coupled ($\sigma = 0.01$) and the control gains q_i are all equal to 40.

As we can see from Figure 1, at the onset of the evolution, the snapping dynamics initiates and the pinning error is suddenly reduced; while after time 20, only a small fraction of the network nodes is pinned. Nonetheless, this fraction is not enough to control the network: at time 38 we can observe a burst in the error dynamics. The snapping dynamics is able to immediately react to the sudden increase of the pinning error, deciding to pin another fraction of the network nodes. This leads the network to a stable controlled trajectory.

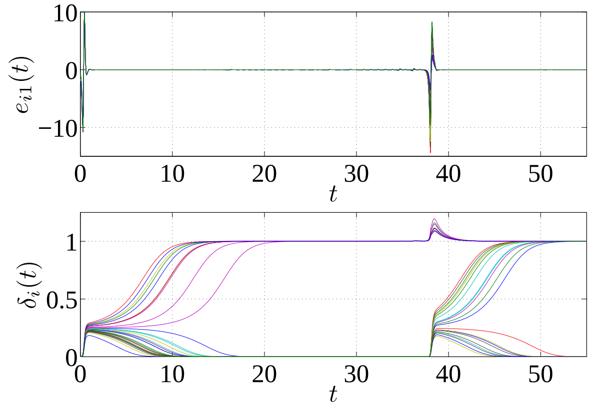

Fig. 1. Pinning control via edge snapping in a network of 30 Lorenz oscillators with fixed control gains. Evolution of the first component of the pinning error (top) and of the δ_i (bottom).

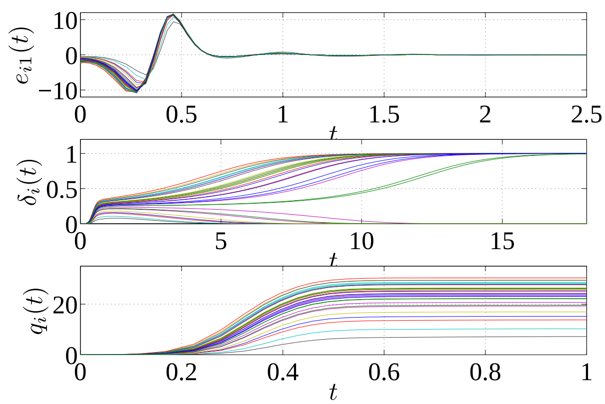

Fig. 2. Pinning control via edge snapping in a network of 30 Lorenz oscillators with adaptive control gains. Evolution of the first component of the pinning error (top), the δ_i (center) and the control gains q_i (bottom).

B. *Adaptive control gains*

The edge snapping dynamics, given the steady-state coupling strength q_i, adjusts the pinning configuration to guarantee the control of the network. Nonetheless, sometimes it is necessary to tune appropriately the control gains. A fine tuning of the coupling may require the knowledge of the network topology, the coupling gain σ, and the number of pinned nodes [13]. To avoid an off-line tuning of the parameters, we propose to tune the control gains as in [21], so that the pinner negotiates with each pinned node the intensity of the corresponding control gain.

To this aim, we consider the following adaptive law for the control gains:

$$\dot{q}_i = e_i^T \Gamma e_i, \tag{10}$$

where Γ is a positive semi-definite matrix. Through these adaptation law, the control gains are not selected a priori, but they are modulated on the basis of the evolution of the pinning error.

To test the effectiveness of these approach, we consider the same network of the previous simulation. The matrix Γ is selected to be equal to $0.01I$, where I is the $n \times n$ diagonal matrix and the control gains are assumed to be initially null. As depicted in Figure 2, the convergence to the desired trajectory is considerably faster than that obtained with a fixed coupling gain. Moreover, the steady-state control gains are in average 23.1 and even the maximum value is lower than the fixed control gain used in the simulation illustrated in Figure 1. Therefore, the adaptation of the coupling gains seem to improve the performance of the pinning scheme while reducing the control effort.

III. Fully Decentralized Edge Snapping Control

In the classical network equation (2), the coupling gain σ is supposed to be constant and equal for all nodes/edges in the network. This model may be inadequate to describe the controlled dynamics of a real network. In fact, real-world networks are often characterized instead by evolving and adapting couplings which vary in time according to different environmental conditions. Such networks include wireless networks of sensors that gather and communicate data to a central base station [22], [23]. In these cases, it is realistic to assume that the strength of the interactions among nodes, characterized mathematically by σ, is not identical for every node and time-invariant. In addition, the network topology itself needs to be adaptively updated on the basis of the dynamic evolution of the network. For these reasons, we propose to modify the control scheme presented in Section 2 by introducing the fully decentralized edge snapping control scheme:

$$\begin{aligned}
\dot{x}_i &= f(x_i,t) + \sum_{j=1}^{N} a_{ij}(t)\sigma_{ij}(t)(h(x_j) - h(x_i)) + \\
&\quad \delta_i q_i (h(x_j) - h(x_i)), \qquad i \in \mathcal{V}, \quad (11)\\
\dot{x}_s &= f(x_s,t) \qquad (12)\\
\ddot{\delta}_i &= -\zeta\dot{\delta}_i - \frac{d}{d\delta_i}V(\delta_i) + g(e_i), \qquad i \in \mathcal{V}, \quad (13)\\
\ddot{a}_{ij} &= -\xi\dot{a}_{ij} - \frac{d}{da_{ij}}V(a_{ij}) + c(e_{ij}), \quad (i,j) \in \mathcal{E}, \quad (14)\\
\dot{q}_i &= e_i^T \Gamma e_i, \qquad i \in \mathcal{V}, \quad (15)\\
\dot{\sigma}_{ij} &= e_{ij}^T \Gamma e_{ij} \qquad (i,j) \in \mathcal{E}. \quad (16)
\end{aligned}$$

Here, (11) and (12) describe the dynamics of the network nodes and of the pinner; (13) and (14) are the snapping dynamics determining respectively the network topology and the set of pinned nodes; and (15) and (16) allow for the modulation of the control and coupling gains.

The main advantage of this approach is that every aspect of the control is adapted in a decentralized way: each pair of nodes in the controlled network can negotiate to decide weather activating or not their corresponding link, and what should be the intensity of the coupling. In addition, the pinner adaptively select the pinned nodes and the corresponding control gains.

As a numerical example, we consider the same network analyzed in Figures 1 and 2 and we set $\sigma_{ij}(0) = 0$. By comparing Figures 2 and 3, we evince that the convergence to the desired trajectory is slightly faster with the fully decentralized approach, notwithstanding that the network is disconnected at the onset of the evolution. Figure 4 shows that all the coupling and control gains converge to steady-state values. Figure 5 illustrates the adaptive selection of the network edges and the pinned nodes.

IV. Conclusions

In this paper, we presented novel pinning control scheme to avoid the need of an off-line tuning of the pinning parameters. Differently from previous works, such as [11], [13], [21], [24], in which only the control gains are adaptively updated, we proposed an adaptive scheme for the selection of the pinned nodes as well. Namely, we proposed to use the so-called edge snapping mechanism to decide pinned nodes.

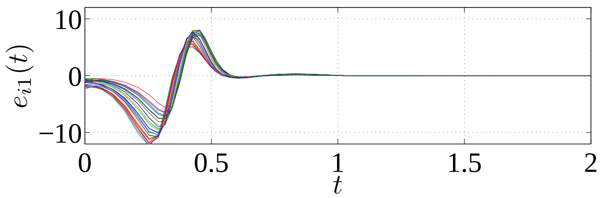

Fig. 3. Network of 30 Lorenz oscillators with fully decentralized snapping control. Evolution of the first component of the pinning error.

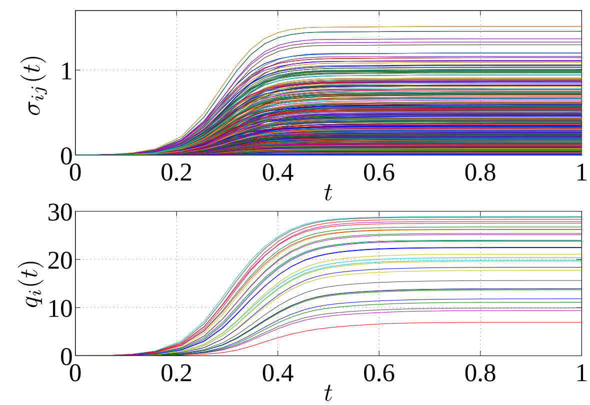

Fig. 4. Network of 30 Lorenz oscillators with fully decentralized snapping control. Evolution of the coupling (top) and control gains (bottom).

We numerically assessed effectiveness of this approach with fixed and adaptive control gains, highlighting the strong improvement of the performances when the control gains are adapted.

Then, we proposed the fully decentralized snapping control scheme, that implements the adaptive pinning selection and gain modulation to an evolving network. This scheme is able to adaptively determine its steady-state topology and the strength of the interaction between each nodes'pair. This approach shows superior performance if compared with the static network. This result is somehow surprising if we consider that the network at the onset of the evolution is

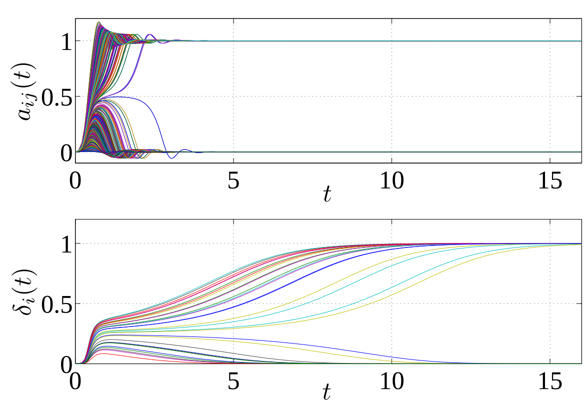

Fig. 5. Network of 30 Lorenz oscillators with fully decentralized snapping control. Evolution of a_{ij} (top) and δ_i (bottom).

disconnected.

The pinning control scheme proposed in this paper seems to be a promising approach for overcoming the limitations of classical pinning control using an adaptive approach. Future work include the analysis of the stability properties of this control scheme, to better assess its full potential.

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