H_∞ Observers design for a class of continuous nonlinear singular systems

M. Darouach, L. Boutat-Baddas and M. Zerrougui

***Abstract*— This paper presents a new solution to the H_∞ observers design problem for a class of Lipschitz continuous nonlinear singular systems. The approach is based on the parameterization of the solution of generalized Sylvester equations. Sufficient conditions for the existence of the observers which guarantee stability and the worst case observers error energy over all bounded energy disturbances is minimized are given. The method also concerns the full-order, reduced-order, minimal-order observers design A numerical example is given to show the applicability of our results.**

keywords: H_∞ Observers design, reduced order, minimal order, full order, nonlinear singular systems, LMI, continuous time systems.

I. INTRODUCTION

The observers design for nonlinear systems has been a very active field during the last two decades. This is due to the fact that a state estimation is generally required for the control when all states of the system are not available. The observers are also used in the monitoring and fault diagnosis. For standard nonlinear systems there exist several approaches for the observers design including one based on coordinate transformations which lead to a linear error dynamics ([7], [8], [9]) and one where the problem of the observers design can be treated without the need of these transformations [10]. An important class of standard nonlinear systems, the global Lipschitz, was considered by [4] and [5], where the existence conditions for the observers are presented and constructive design methods were given for full-order and reduced-order cases.

On the other hand singular systems (known as generalized, descriptor or differential algebraic (DA) systems) describe a large class of systems. They are encountered in chemical and mineral industries, for example the dynamic balances of mass and energy are described by differential equations while thermodynamic equilibrium relations constitute additional algebraic constraints. The problem of the state estimation for these practical applications arises in data reconciliation for example [1]. Singular systems are also frequently encountered in electronic and economics [12]. In recent years a great deal of work has been devoted to the analysis and design techniques for singular systems [2] and [3]. On the other hand, the problem of observer design for linear systems has been greatly treated for the standard and singular systems with or without unknown inputs (see [6], [21], [11]and references therein). In [13], extension to observers design for Lipschitz singular systems has been presented, however the observer considered has a singular system form. Recently a new method for the observers design is presented for a class of singular systems, where the nonlinearity is assumed to be composed of Lipschitz one and an arbitrary one, the latter can be considered as an unknown disturbance. The approach is based on the parameterization of the generalized Sylvester equations solutions and unifies the design of full, reduced and minimal orders observers, the observer presented is causal and has a standard system form [14]. However, only the case where the model and the measurement are free from noises.

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The state estimation problem for linear singular systems in presence of noises has been the subject of several studies in the past decades. We can distinguish two approaches, the Kalman filtering approach and H_∞ approach. In the Kalman filtering, the system and the measurement noises are assumed to be Gaussian with known statistics [15], [16], [17]. When the noises are arbitrary signals with bounded energy, the H_∞ filtering permits to guarantee a noise attenuation level [18]. Recently, a number of papers have appeared that deal with the H_∞ filtering for singular systems, see for example [19], [20], [22] and references therein. In all these works only full or reduced order filters were presented for the square singular systems.

In this paper, we consider the H_∞ observers design problem for a class of Lipschitz nonlinear singular systems. The approach is based on the work [14] and considers the case where the model and the measurement are affected by noises. Sufficient conditions in terms of LMIs are given for this H_∞ problem. The method unifies the design for full-order, reduced-order and minimal order observers. A numerical example is given to illustrate our results.

II. PROBLEM FORMULATION

Consider the following nonlinear singular system

$$E\dot{x}(t) = Ax + Bu + Df(t, F_L x, u) + D_1 w \quad (1a)$$
$$y(t) = Cx(t) + D_2 w(t) \quad (1b)$$

with the initial state $x(0) = x_0$. Where $x(t) \in I\!R^n$ is the semi state vector, $u(t) \in I\!R^m$ is the known input, $w(t) \in I\!R^{n_w}$ is the disturbance vector containing both system and measurement noises, and $y(t) \in I\!R^p$ is the measurement output. Matrix $E \in I\!R^{n_1 \times n}$ and when $n_1 = n$ matrix E is singular. Matrices $A \in I\!R^{n_1\times n}$, $B \in I\!R^{n_1\times m}$, $C \in I\!R^{p\times n}$, $D \in I\!R^{n_1\times n_f}$, $D_1 \in I\!R^{n_1\times n_w}$ and $D_2 \in I\!R^{p\times n_w}$. The nonlinearity $f(t, F_L x, u)$ verifies the Lipschitz constraints:

$$\|f(t, F_L x_1, u) - f(t, F_L x_2, u) \le \lambda(\|F_L(x_1 - x_2)\|),$$

where λ is a known Lipschitz constant, matrix F_L is real with appropriate dimension.

Now as in [14], let $\Phi \in I\!R^{r_1 \times n_1}$ be a full row rank matrix such that

$$\Phi \begin{bmatrix} E & D \end{bmatrix} = 0.$$

then, from (1) we obtain

$$\Phi A x(t) + \Phi D_1 w(t) = -\Phi B u(t).$$

In the sequel we assume that

Assumption 1: $rank \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix} = n.$

Assumption 1 is exactly the impulse observability of the linear singular system (E, A, B, C).

Now, let us consider the following reduced-order observer for system (1)

$$\begin{aligned} \dot{\zeta}(t) &= N\zeta(t) + Jy(t) + Hu(t) + TDf(t, F_L\widehat{x}, u) && (2a) \\ \widehat{x}(t) &= P\zeta(t) - Q\Phi Bu(t) + Fy(t) && (2b) \end{aligned}$$

with the initial condition $\zeta(0) = \zeta_0$. Vector $\zeta(t) \in \mathbb{R}^q$ represents the state vector of the observer and $\widehat{x}(t) \in I\!R^n$ is the estimate of $x(t)$. Matrices N, J, T, H, P, Q, and F are unknown matrices of appropriate dimensions, which must be determined such that:

1- for $w(t) = 0$, the error $e(t) = \widehat{x}(t) - x(t)$ asymptotically converges to 0.

2- for $w(t) \neq 0$ we solve the $\min \sup_{\omega \in L_2 - \{0\}} \frac{\|e\|_{L_2}}{\|\omega\|_{L_2}}$.

Let the error between $\zeta(t)$ and $TEx(t)$ be

$$\epsilon(t) = \zeta(t) - TEx(t)$$

then we obtain the following dynamics of $\epsilon(t)$

$$\begin{aligned} \dot{\epsilon}(t) &= \dot{\zeta} - TE\dot{x} = N\epsilon + (NTE - TA + JC)x && (3) \\ &+ (H - TB)u(t) + TD\Delta f + (JD_2 - TD_1)w(t) \end{aligned}$$

where $\Delta f = f(t, F_L\widehat{x}(t), u) - f(t, F_L x(t), u)$. On the other hand from (2b) and the definition of ϵ, we have

$$\widehat{x}(t) = P\epsilon + \begin{bmatrix} P & Q & F \end{bmatrix} \begin{bmatrix} TE \\ \Phi A \\ C \end{bmatrix} x + (Q\Phi D_1 + FD_2)w \quad (4)$$

Under Assumption 1, if there exists a matrix T such that

$$\begin{aligned} NTE - TA + JC &= 0 && (5a) \\ H &= TB && (5b) \\ \begin{bmatrix} P & Q & F \end{bmatrix} \begin{bmatrix} TE \\ \Phi A \\ C \end{bmatrix} &= I && (5c) \end{aligned}$$

then equations (3) and (4) become

$$\begin{aligned} \dot{\epsilon}(t) &= N\epsilon(t) + TD\Delta f + (JD_2 - TD_1)w(t) && (6a) \\ e(t) &= P\epsilon(t) + (Q\Phi D_1 + FD_2)w(t) && (6b) \end{aligned}$$

Now the problem of the H_∞ observer design is reduced to find the matrices N, J, H, P, Q, F and T such that (5) are satisfied and the worst case observers error energy over all bounded energy disturbances $w(t)$ is minimized.

III. Main results

In this section we shall give the sufficient conditions for the H_∞ observers design.

Let $\varphi_1 = JD_2 - TD_1$ and $\varphi_2 = Q\Phi D_1 + FD_2$, then equation (6) can be written as

$$\begin{aligned} \dot{\epsilon}(t) &= N\epsilon(t) + TD\Delta f + \varphi_1 w(t) && (7a) \\ e(t) &= P\epsilon(t) + \varphi_2 w(t) && (7b) \end{aligned}$$

The H_∞ observer design problem presented can be formulated as follows: given the nonlinear singular system (1) and a prescribed level of noise $\gamma > 0$, find a suitable observer in the form (2), such that:

1) The observer error (7) with $w(t) = 0$ is stable.

2) Under zero initial condition, the induced L_2 norm of the operator from $w(t)$ to $e(t)$ is less than γ, i.e $\|e\|_{L^2} < \gamma\|w\|_{L^2}$

1) Stability analysis: One can see that the asymptotic stability of $\varepsilon(t)$ is sufficient for $\lim_{t\to\infty} e(t) = 0$, since for $w(t) = 0$ we have $e(t) = P\varepsilon(t)$. The following lemma gives the conditions of the stability of $e(t)$.

Now we can give the following lemma

Lemma 1: Under Assumption 1, for $w(t) = 0$, the error dynamics (7) is asymptotically stable, if there exist a positive definite matrix $X > 0$, matrices P and T and a positive scalar μ such that

$$\begin{bmatrix} N^T X + XN + \lambda^2 \mu P^T F_L^T F_L P & XTD \\ (TD)^T X & -\mu I \end{bmatrix} < 0 \quad (8)$$

Proof: Let the Lyapunov candidate function be

$$V(t) = \varepsilon^T(t) X \varepsilon(t),$$

where X positive definite matrix. Then the derivative of $V(t)$ along the solution of (7) is given by

$$\begin{aligned} \dot{V}(t) &= \dot{\varepsilon}^T X \varepsilon + \varepsilon^T X \dot{\varepsilon} \\ &= [N\varepsilon + TD\Delta f]^T X\varepsilon(t) + \varepsilon^T X [N\varepsilon + TD\Delta f] \\ &= \varepsilon^T \left[N^T X + XN\right] \varepsilon + \Delta f^T (TD)^T X\varepsilon + \varepsilon^T XTD\Delta f \end{aligned}$$

Let u and v be two vectors of appropriate dimensions, then for all scalar $\mu > 0$ the following inequality holds

$$u^T v + v^T u \leqslant \mu u^T u + \frac{1}{\mu} v^T v \quad (9)$$

From (9) we have the following inequality

$$\begin{aligned} \Delta f^T (TD)^T X\varepsilon + \varepsilon^T X (TD) \Delta f &\leqslant \frac{1}{\mu} \varepsilon^T X (TD)(TD)^T X\varepsilon \\ &+ \mu \Delta f^T \Delta f && (10) \end{aligned}$$

Then from the expression of $\dot{V}$ and the fact that $\Delta f^T \Delta f \leqslant \lambda^2 \varepsilon^T P^T F_L^T F_L P\varepsilon$, we obtain$\dot{V}(t) < 0$, if

$$\varepsilon^T [N^T X + XN + \lambda^2 \mu P^T F_L^T F_L P + \frac{1}{\mu} X(TD)(TD)^T X] \varepsilon < 0$$

which is satisfied if

$$N^T X + XN + \lambda^2 \mu P^T F_L^T F_L P + \frac{1}{\mu} X(TD)(TD)^T X < 0$$

By using the Schur complement we obtain (8). Then the lemma is proved. ■

2) H_∞ observes design: In this section we shall present the H_∞ observer design. The following lemma gives the sufficient conditions for system (7) to be stable for $w(t) = 0$ and $\|e(t)\|_{L_2} < \gamma \|w(t)\|_{L_2}$ for $w(t) \neq 0$.

Lemma 2: Under Assumption 1, the error $e(t)$ given by (7) is asymptotically stable for $w(t) = 0$ and $\|e(t)\|_{L_2} < \gamma \|w(t)\|_{L_2}$, if there exist a symmetric positive definite matrix X, matrices T, N, P, φ_1 and φ_2 and a positive scalar μ such that

$$\Sigma^1 = \begin{bmatrix} \Sigma_{11}^1 & X(TD) & \left(\Sigma_{31}^1\right)^T \\ (TD)^T X & -\mu I & 0 \\ \Sigma_{31}^1 & 0 & \varphi_2^T \rho_1 \varphi_2 - \gamma^2 I \end{bmatrix} < 0 \quad (11)$$

where

$$\begin{aligned} \rho_1 &= I + \lambda^2(\mu+1) F_L^T F_L, \\ \Sigma_{11}^1 &= N^T X + XN + P^T \rho_1 P, \\ \Sigma_{31}^1 &= \varphi_1^T X + \varphi_2^T \rho_1 P \end{aligned}$$

Proof: First, we can see that if $\Sigma^1 < 0$, then by Schur complement we obtain LMI (8) and we have the stability for $w(t) = 0$. Now, let $w(t) \neq 0$, in this case we obtain

$$\begin{aligned} &\dot{V}(t) = \dot{\varepsilon}^T(t) X \varepsilon(t) + \varepsilon^T(t) X \dot{\varepsilon}(t) \\ &= [N\varepsilon + TD\Delta f + \varphi_1 w]^T X \varepsilon + \varepsilon^T X [N\varepsilon + TD\Delta f + \\ &\varphi_1 w] = \varepsilon^T [N^T X + XN] \varepsilon + \Delta f^T TDX\varepsilon + \\ &\varepsilon^T X(TD)\Delta f + w^T \varphi_1^T X \varepsilon + \varepsilon^T X \varphi_1 w \end{aligned}$$

. As in the proof of Lemma 1, by using inequality (10) and the expression of $\dot{V}$ we obtain

$$\begin{aligned} \dot{V} &\leqslant \varepsilon^T [N^T X + XN + \frac{1}{\mu} X(TD)(TD)^T X] \varepsilon + \mu \Delta f^T \Delta f \\ &\quad + w^T \varphi_1^T X \varepsilon + \varepsilon^T X \varphi_1 w \\ &\leqslant \varepsilon^T [N^T X + XN + \frac{1}{\mu} X(TD)(TD)^T Xt] \varepsilon \\ &\quad + \mu \lambda^2 e^T(t) F_L^T F_L e(t) + w^T \varphi_1^T X \varepsilon + \varepsilon^T X \varphi_1 w \end{aligned}$$

Where we have used the fact that $\Delta f^T \Delta f \leqslant \lambda^2 e^T F_L^T F_L e$.

Define $\tau = \begin{bmatrix} \varepsilon \\ w \end{bmatrix}$, then we obtain

$$\begin{aligned} &\dot{V} + e^T e - \gamma^2 w^T w \leqslant \\ &\tau^T \begin{bmatrix} \bar{\Sigma} & X\varphi_1 + P^T \rho_1 \varphi_2 \\ \varphi_1^T X + \varphi_2^T \rho_1 P & \varphi_2^T \rho_1 \varphi_2 - \gamma^2 I \end{bmatrix} \tau, \end{aligned}$$

where $\bar{\Sigma} = N^T X + XN + P^T \rho_1 P + \frac{1}{\mu} X(TD)(TD)^T X$.

If (11) is satisfied, by using the Schur complement, we obtain

$$\dot{V} < \gamma^2 w^T w - e^T e$$

under zero initial conditions, we obtain

$$V(\infty) < \gamma^2 \|w(t)\|_2^2 - \|e(t)\|_2^2$$

which leads to

$$\|e(t)\|_2^2 < \gamma^2 \|w(t)\|_2^2$$

This completes the proof of the lemma. ■

Before giving the design method for the observer (2), let us consider equations (5) and let $\tilde{T} = T + \Psi\Phi$, where Ψ is an arbitrary matrix of appropriate dimension, they can be written as

$$\begin{bmatrix} N & \Psi & J \\ P & Q & F \end{bmatrix} \begin{bmatrix} \tilde{T}E \\ \Phi A \\ C \end{bmatrix} = \begin{bmatrix} \tilde{T}A \\ I_n \end{bmatrix} \quad (12)$$

Equation (12) has a solution if and only if

$$rank \begin{bmatrix} \tilde{T}E \\ \Phi A \\ C \\ \tilde{T}A \\ I_n \end{bmatrix} = rank \begin{bmatrix} \tilde{T}E \\ \Phi A \\ C \end{bmatrix} = n \quad (13)$$

Now, from Assumption 1 and equation (13), we have

$$rank \begin{bmatrix} \tilde{T}E \\ \Phi A \\ C \end{bmatrix} = \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix} = n$$

Let R be any full row rank matrix such that

$$rank \begin{bmatrix} R \\ \Phi A \\ C \end{bmatrix} = rank \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix} = rank \begin{bmatrix} \tilde{T}E \\ \Phi A \\ C \end{bmatrix} = n$$

then there always exist matrices parameter K and $\tilde{T}$ such that

$$\tilde{T}E = R - K \begin{bmatrix} \Phi A \\ C \end{bmatrix} \quad (14)$$

or equivalently

$$\begin{bmatrix} \tilde{T} & K \end{bmatrix} \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix} = R$$

Then under Assumption 1, there exists a solution to (14) given by

$$\begin{bmatrix} \tilde{T} & K \end{bmatrix} = R \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix}^{\dagger} \quad (15)$$

In this case matrices $\tilde{T}$ and K are given by

$$\tilde{T} = R \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix}^{\dagger} \begin{bmatrix} I \\ 0 \end{bmatrix}$$

and

$$K = R \begin{bmatrix} E \\ \Phi A \\ C \end{bmatrix}^{\dagger} \begin{bmatrix} 0 \\ I \end{bmatrix}$$

Also, under Assumption 1, the general solution to (12) is given by

$$\begin{bmatrix} N & \Psi & J \\ P & Q & F \end{bmatrix} = \begin{bmatrix} \tilde{T}A \\ I_n \end{bmatrix} \Omega^{\dagger} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \left(I - \Omega\Omega^{\dagger}\right) \quad (16)$$

here $\Omega = \begin{bmatrix} \tilde{T}E \\ \Phi A \\ C \end{bmatrix}$ and $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ is an arbitrary matrix of appropriate dimension. Now, define the following matrices

$$\begin{aligned} \Lambda_P &= \Omega^{\dagger} \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}, \Lambda_Q = \Omega^{\dagger} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \\ \Lambda_N &= \tilde{T}A\Lambda_P, \Lambda_\Psi = \tilde{T}A\Lambda_Q, \Lambda_J = \tilde{T}A\Lambda_F, \\ \Delta_N &= (I - \Omega\Omega^{\dagger}) \begin{bmatrix} I \\ 0 \\ 0 \end{bmatrix}, \Delta_\Psi = (I - \Omega\Omega^{\dagger}) \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, \\ \Delta_J &= (I - \Omega\Omega^{\dagger}) \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \Lambda_F = \Omega^{\dagger} \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \end{aligned}$$

then we obtain

$$\begin{array}{rcl} N &=& \Lambda_N - Y_1\Delta_N, \Psi = \Lambda_\Psi - Y_1\Delta_\Psi \\ J &=& \Lambda_J - Y_1\Delta_J, P = \Lambda_P - Y_2\Delta_N \\ Q &=& \Lambda_Q - Y_2\Delta_\Psi, F = \Lambda_F - Y_2\Delta_J \end{array}$$

From these values we can deduce the expressions of φ_1, φ_2 and TD as follows $\varphi_1 = \Lambda_{\varphi_1} - Y_1\Delta_{\varphi_1}$, $\varphi_2 = \Lambda_{\varphi_2} - Y_2\Delta_{\varphi_2}$, and $TD = \Lambda_{TD} - Y_1\Delta_{TD}$ where $\Lambda_{\varphi_1} = \left(\Lambda_J D_2 - \tilde{T}D_1 - \Lambda_\Psi \Phi D_1\right)$, $\Delta_{\varphi_1} = (\Delta_J D_2 - \Delta_\Psi \Phi D_1)$, $\Delta_{\varphi_2} = (\Delta_Q \Phi D_1 + \Delta_J D_2), \Lambda_{\varphi_2} = (\Lambda_Q \Phi D_1 + \Lambda_F D_2)$, $\Lambda_{TD} = \tilde{T}D + \Lambda_\Psi \Phi D$, and $\Delta_{TD} = \Delta_\Psi \Phi D$.

Now, the H_∞ observer design can be obtained from the following theorem.

Theorem 1: Under Assumption 1, there exists an observer of the form (2) such that the error $e(t)$ given by (7) is asymptotically stable for $w(t) = 0$ and $\|e(t)\|_{L_2} < \gamma \|w(t)\|_{L_2}$ if there exist a symmetric positive definite matrix X, matrices Ω_{Y_1} and $\bar{Y}_2$ and a positive scalar μ such that the following LMI-LME are satisfied

$$\begin{bmatrix} (1,1) & (2,1)^T & (3,1)^T & (4,1)^T \\ (4,1) & -\mu I & 0 & 0 \\ (3,1) & 0 & -\gamma^2 I & (4,3)^T \\ (4,1) & 0 & (4,3) & -\rho \end{bmatrix} < 0 \qquad (17)$$

and

$$\rho_1 = I + \lambda^2(\mu+1)F_L^T F_L \qquad (18)$$

where

$$\begin{array}{l} (1,1) = \Lambda_N^T X + X\Lambda_N - \Delta_N^T \Omega_{Y_1}^T - \Omega_{Y_1}\Delta_N. \\ (2,1) = \Lambda_{TD}^T X - \Delta_{TD}^T \Omega_{Y_1}^T \\ (3,1) = \Lambda_{\varphi_1}^T X - \Delta_{\varphi_1}^T \Omega_{Y_1}^T \\ (4,1) = \rho\Lambda_P - \bar{Y}_2\Delta_N \\ (4,3) = \rho\Lambda_{\varphi_2} - \bar{Y}_2\Delta_{\varphi_2} \\ Y_1 = X^{-1}\Omega_{Y_1} \text{and} Y_2 = \rho_1^{-1}\bar{Y}_2. \end{array}$$

Proof: Under Assumption 1, from Lemma 2 the error $e(t)$ given by (7) is asymptotically stable for $w(t) = 0$ and $\|e(t)\|_{L_2} < \gamma \|w(t)\|_{L_2}$, if there exist a symmetric positive definite matrix X, matrices T, N, P, φ_1, φ_2 and a positive scalar μ such that

$$\Sigma_{11}^1 = \begin{bmatrix} \Sigma_{11}^1 & X(TD) & \left(\Sigma_{31}^1\right)^T \\ (TD)^T X & -\mu I & 0 \\ \Sigma_{31}^1 & 0 & \varphi_2^T\rho_1\varphi_2 - \gamma^2 I \end{bmatrix} < 0 \qquad (19)$$

where

$$\begin{array}{l} \rho_1 = I + \lambda^2(\mu+1)F_L^T F_L, \\ \Sigma_{11}^1 = N^T X + XN + P^T\rho_1 P, \\ \Sigma_{31}^1 = \varphi_1^T X + \varphi_2^T \rho_1 P \end{array}$$

and by using the Schur complement we obtain

$$\begin{bmatrix} N^T X + XN & X(TD) & X\varphi_1 & P^T\rho_1 \\ (TD)^T X & -\mu I & 0 & 0 \\ \varphi_1^T X & 0 & -\gamma^2 I & \varphi_2^T \rho_1 \\ \rho_1 P & 0 & \rho_1\varphi_2 & -\rho_1 \end{bmatrix} < 0 \qquad (20)$$

Inserting the values of N, P, TD, φ_1 and φ_2 into (20), leads to the LMI (17). Set $Y_1 = X^{-1}\Omega_{Y_1}$ and $Y_2 = \rho_1^{-1}\bar{Y_2}$. Which proves the theorem. ■

Remark 1: When the value of the scalar parameter μ is fixed, the problem is reduced to solve the LMI (17). Now, to solve the LMI-LME given by (17) and (18). One can set $\mu = \frac{\alpha}{1-\alpha}$, where α is a positive parameter such that $\alpha \in]0,1[$, this is equivalent to $\mu > 0$. In this case the parameter

$$\rho_1 = I + \lambda^2(\mu+1)F_L^T F_L = I + \frac{1}{1-\alpha}\lambda^2 F_L^T F_L.$$

The problem is reduced to solve the LMI (17) for $\alpha \in]0,1[$.

IV. Numerical example

Let us consider the following continuous nonlinear singular system of the form (1) with

$$E = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \; D = \begin{bmatrix} 1 \\ 1 \\ 0.1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$D_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ and $u(t) = \sin(2t)$. The nonlinearity $f(x,u,t) = sin(x_3(t))$. For this system, the matrix $\Phi = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$. In this case it is easy to see that Assumption 1 is verified. We shall design a reduced-order observer of dimension $q = 2$, let $R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $rank \begin{bmatrix} R \\ \Phi A \\ C \end{bmatrix} = 3$. For $\gamma^2 = 0.01$, we obtain from section 3 the following results:

$\Omega_{Y_1} = 10^2 \begin{bmatrix} -2.167 & 0.221 & 0.150 & -0.185 \\ -0.249 & -0.761 & -0.498 & 0.650 \end{bmatrix}$ and $X = \begin{bmatrix} 0.376 & -0.257 \\ -0.257 & 0.210 \end{bmatrix}$. Then in this case the H_∞ observer is given by the following model:

$$\begin{array}{l} \dot{\zeta}(t) = \begin{bmatrix} 0 & -2.297 \\ 0.666 & -3.349 \end{bmatrix} \zeta(t) + \begin{bmatrix} -0.531 \\ -2.233 \end{bmatrix} y(t) \\ \quad + \begin{bmatrix} -0.531 \\ -1.566 \end{bmatrix} u(t) + \begin{bmatrix} 0.984 \\ 0.644 \end{bmatrix} sin(\widehat{x}_3(t)) \\ \widehat{x}(t) = \begin{bmatrix} 0 & 0.233 \\ 1 & -0.071 \\ 0 & 0 \end{bmatrix} \zeta(t) - \begin{bmatrix} -0.155 \\ 0.047 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1.155 \\ -0.047 \\ 1 \end{bmatrix} y(t) \end{array} \qquad (21)$$

Simulation results are presented in figures 1-4. Figure 1 presents the noises $w_1(t)$ and $w_2(t)$. Figures 2 -4 show the estimation of the states x_1, x_2 and x_3. They show the good performances of our results.

V. Conclusion

In this paper a new method for the H_∞ observers design for a class of Lipschitz continuous nonlinear singular systems has been developed. The obtained results unify the observers design of full, reduced and minimal orders. Sufficient conditions for the existence of these observers are given in terms of LMIs. A numerical example was given to show the applicability of our approach. The extension of our work to more general nonlinear singular systems is under study.

REFERENCES

[1] M.Darouach and M.Zasadzinski. Data reconciliation in generalized linear dynamic systems. *AICHE journal*, vol.37, Issue 2, pp.193-201, 1991.

[2] S.L.Campbell. *Singular systems of differential equations.* Vol.I and vol.II: Pitman, 1980, 1982.

[3] L.Dai. *Singular control systems* . New York: Springer-Verlag, 1989.

[4] R.Rajamani. Observers for Lipschitz nonlinear systems. *IEEE Trans.Autom.Contr* , vol.Ac-43, pp.397-401, 1998.

[5] F.Zhu and Z.Han. A note on observers for Lipschitz nonlinear systems. *IEEE.Trans.Automat.Contr.* , vol 47, pp.1751-1754, 2002.

[6] M.Darouach, M.Zasadzinski, and S.J.Xu. Full-order observers for linear systems with unknown inputs. *IEEE.Trans.Automat.Contr.* , vol 39, pp.606-609, 1994.

[7] J.Krener and W.Respondek. Nonlinear observer design with linearizable error dynamics. *SIAM Journal on Control and Optimization*, vol.23, no 2, pp.197-216, 1985.

[8] H.Keller. Nonlinear observer design by transformation into generalized observer canonical form. *Int.J.Control*, vol.46, no 6, pp.1915-1930, 1987.

[9] X.H.Xiao and W.Gao. Nonlinear observer design by observer error linearization. *SIAM Journal on Control and Optimization*, vol.27, pp.199-216, 1989.

[10] M.Arcak and P.Kokotovic. Nonlinear observers : a circle criterion design and robustness analysis. *Automatica*, vol.37, no12, pp.1923-1930, 2001.

[11] M.Darouach, M.Zasadzinski and M. Hayar. Reduced-order observer design for descriptor systems with unknown inputs. *IEEE Trans.Autom.Contr* , vol 41, n0 7, pp 1068-1072, 1996.

[12] D.J.Luenberger. Nonlinear descriptor systems. *Journal of Economics Dynamics and Control*, vol 1, pp.219-242, 1979.

[13] G.Lu and D.W.C.Ho. Full-order and reduced-order observer for Lipschitz descriptor systems: The unified LMI approach. *IEEE.Trans on Circuits and systems-II : Express Briefs.* , vol 53, no 7, pp 563-567, 2006.

[14] M.Darouach and L.Boutat-Baddas. Observer for a class of nonlinear singular systems. EEE Trans.Autom.Contr, vol 53, no11, pp 2627-2633, 2008.

[15] M.Darouach and M.Zasadzinski and D.Mehdi. State estimation of stochastic singular linear systems. Int. J. Syst. Sci, vol 24, pp 345-354, 1993.

[16] R. Nikoukhah and A. Willsky and B. Levy. Kalman filtering and Riccati equations for descriptor systems EEE Trans.Autom.Contr,vol 37, no 9, pp 1325-1342, 1992.

[17] R. Nikoukhah and S. Campbell and F. Delebecque. Kalman filtering for general discrete-time systems. EEE Trans.Autom.Contr, vol 34, no10, pp 1829-1839, 1999.

[18] J. Watson and K. Grigoriadis. Optimal unbiased filtering via linear matrix inequalities. Syst. Control Lett, vol 35, pp 111-118, 1998.

[19] S. Xu and J. Lam and Y. Zou. H_∞ filtering for singular systems. EEE Trans.Autom.Contr, vol. 48, no. 12, pp 2217- 2222, 2003.

[20] S. Xu and J. Lam. Reduced-orde H_∞ filtering for singular systems. Syst. Control Lett, vol. 56, pp 48-57, 2007.

[21] M. Darouach and M. Boutayeb. Design of observers for descriptor systems. EEE Trans.Autom.Contr, vol. 40, no. 7, pp 1323-1327, 1995.

[22] M.Darouach. H_∞ Unbiased Filtering for Descriptor Systems via LMI. IEEE Trans.Autom.Contr, vol. 54, no. 8, pp 1966-1972, 2009.

Fig. 1. The noises $w_1(t)$ and $w_2(t)$

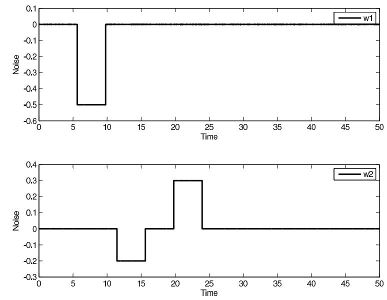

Fig. 2. x_1 (solid lines) and $\hat{x}_1$ (dashed lines)

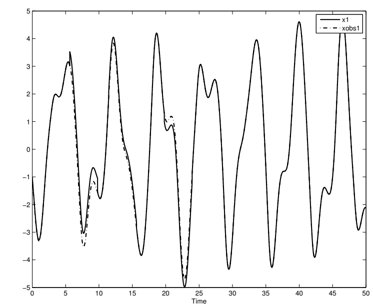

Fig. 3. x_2 (solid lines) and $\hat{x}_2$ (dashed lines)

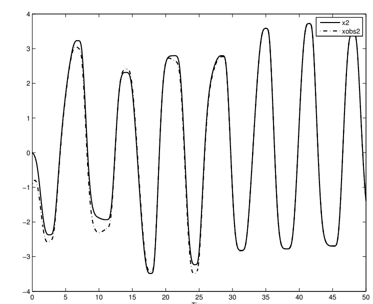

Fig. 4. x_3 (solid lines) and $\hat{x}_3$ (dashed lines)