# Real-time Obstacle Avoidance of a Two-wheeled Mobile Robot via the Minimum Projection Method

Yoshiro Fukui, Hisakazu Nakamura and Hirokazu Nishitani

***Abstract*— This paper considers real-time obstacle avoidance control of a two-wheeled nonholonomic mobile robot. In this paper, we propose a discontinuous asymptotic stabilizing state feedback control law for a real-time obstacle avoidance problem via the minimum projection method. The method guarantees asymptotic stability and reduces the computational cost. The effectiveness of the proposed method is comfirmed by experiments.**

## I. INTRODUCTION

This paper considers real-time obstacle avoidance control of a two-wheeled nonholonomic mobile robot. The robot navigation problem traditionally consists of three steps [2]: path planning, trajectory planning and the control problem. However, stability is only guaranteed in the neighborhood of the designed path, and we require high computational cost for path planning and trajectory planning. Nakamura et al. proposed the minimum projection method [3] that does not need path planning. However, the method does not discuss application to nonholonomic systems like a two-wheeled mobile robot. In this paper, we propose a discontinuous asymptotic stabilizing state feedback control law for a real-time obstacle avoidance problem via the minimum projection method. The method guarantees asymptotic stability and reduces the computational cost. The effectiveness of the proposed method is comfirmed by experiments.

## II. PRELIMINARIES

As a preliminary, we introduce control systems defined on the open subset of Euclidean space and asymptotically stability.

In this paper, $S^1 := \{(x,y)|(x,y) \in \mathbb{R}^2, x^2+y^2=1\}$ is the circle space parametrized by $(-\pi,\pi]$. $X, \tilde{X} \subset \mathbb{R}^2 \times S^1$ and $\mathbb{R}_{\geq 0} := [0,\infty) \subset \mathbb{R}$.

### A. Nonlinear control system

We consider the following nonlinear control system on $X$:

$$\dot{q} = g(q)u \tag{1}$$

where $q \in X, u \in F(\mathbb{R},\mathbb{R}^2); t \mapsto u(t) \in \mathbb{R}^2$, where $F(\mathbb{R},\mathbb{R}^2)$ denotes a set of mappings from $\mathbb{R}$ to $\mathbb{R}^2$. Moreover, $g : X \to \mathbb{R}^{3\times 2}$ is assumed to be continuous with respect to $x$, and the non-empty arc-connected compact subset $\Pi$ of $X$ is a desired set.

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### B. Euler solution

The strictly increasing sequence $\pi : \mathbb{N} \to \mathbb{R}; i \mapsto t_i$ with $\lim_{i\to\infty} t_i = \infty, t_0 = 0$ is called a partition.

Assume given feedback $u : X \to \mathbb{R}^2$, a partition $\pi$ and an initial point $q \in X$. For each $t \in [t_i, t_{i+1}), i \in \mathbb{N}$,

$$\begin{aligned}\psi(q,t) &= (\psi_x(q,t), \psi_y(q,t), \psi_\theta(q,t)) \\ &:= q + (t-t_i)g(\psi(q,t_i))u(\psi(q,t_i)) \\ &\quad + \sum_{j=0}^{i-1}(t_{j+1}-t_j)g(\psi(q,t_j))u(\psi(q,t_j))\end{aligned} \tag{2}$$

is called Euler solution for (1). We will replace $\psi(q,t)$ by $\psi(t)$ if there is no confusion.

### C. Asymptotic stability[6]

The limit set $\Lambda^+ : X \to 2^X$ is defined as follows:

$$\Lambda^+(q) := \{q_\omega | q_\omega \in X, \text{there is a sequence } \lambda : \mathbb{N} \to \mathbb{R}; n \mapsto t'_n \text{ with } \lim_{n\to\infty} t'_n = \infty \text{ and } \lim_{n\to\infty} \psi(q,t'_n) = q_\omega\}. \tag{3}$$

The domain of attraction $A : 2^X \to 2^X$ defined as follows:

$$A(\Pi) := \{q | q \in X, \Lambda^+(q) \neq \emptyset, \Lambda^+(q) \subset \Pi\}, \tag{4}$$

where $2^X$ denots a power set of $X$ and $\Pi$ is the desired compact subset of $X$.

A set $\Pi$ is called positively invariant whenever $\psi(q,t) \in \Pi$ for all $q \in \Pi, t \in \mathbb{R}_{\geq 0}$.

A set $\Pi$ is said to be a global attractor, if $A(\Pi) = X$.

A set $\Pi$ is said to be stable if every neighborhood $U$ of $\Pi$ contains a positively invariant neighborhood $V$ of $\Pi$.

A set $\Pi$ is said to be global asymptotically stable if it is stable and a global attractor.

**Remark 1** *If $\Pi$ is a singleton $\{q_\omega\}$ and $\lim_{t\to\infty} \psi(q,t) = q_\omega$ for all $q \in X$, $\Pi$ is global asymptotically stable.*

## III. PROBLEM STATEMENT

In this paper, we consider the following two-wheeled mobile robot system:

$$\dot{q} = g(q)u, \tag{5}$$

where

$$q := \begin{bmatrix} q_x \\ q_y \\ q_\theta \end{bmatrix} \in X, g(q) := \begin{bmatrix} \cos q_\theta & 0 \\ \sin q_\theta & 0 \\ 0 & 1 \end{bmatrix}, u := \begin{bmatrix} u_v \\ u_\omega \end{bmatrix}, \tag{6}$$

where $q_x, q_y$ are the Cartesian coordinates of the center of the robot, $q_\theta$ is the angle between the heading direction and the $x$-axis, $u_v$ denotes the linear velocity and $u_\omega$ is the angular velocity of the mobile robot.

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We consider the following configuration space having an obstacle:

$$X := (\mathbb{R}^2 \backslash D) \times S^1 \tag{7}$$
$$= \{(q_x, q_y, q_\theta) | (q_x + p_d)^2 + q_y^2 > r_d, q_\theta \in \mathbb{R}\}, \tag{8}$$

where $D$ is a disc-shaped obstacle with radius $r_d > 0$ centered at $(-p_d, 0)$ satisfying $p_d > r_d$.

The problem is global asymptotic stabilization of the following desired set $\Pi$ [5]:

$$\Pi := \{(q_x, q_y, q_\theta) | q_x = q_y = 0, q_\theta \in S^1\}. \tag{9}$$

## IV. Minimum Projection Method

In this paper, we use a potential function on $X$ designed by the minimum projection method for the problem in this paper.The minimum projection method is summarized as follows:

1) Consider a simple-structured space $\mathbb{R}^2 \times S^1$ and choose a $C^1$ surjection $\phi : \mathbb{R}^2 \times S^1 \to X$ such that $|\partial\phi/\partial\tilde{q}| \neq 0$ for all $\tilde{q} \in \mathbb{R}^3$ and $\phi(0) = 0$.
2) Design a potential function $\tilde{V}$ on $\mathbb{R}^3$ for asymptotic stabilization of the origin of the control system $\dot{\tilde{q}} = (\partial\phi/\partial\tilde{q})^{-1} g(\phi(\tilde{q}))u$.
3) The following function is a potential function on $X$ for obstacle avoidance:

$$V(q) = \min_{\tilde{q} \in \phi^{-1}(q)} \tilde{V}(\tilde{q}). \tag{10}$$

When we use the minimum projection method, we can design a potential function $V$ on $X$ from the function $\tilde{V}$ on the simple-structured space $\mathbb{R}^3$. However, the method does not discuss controlling a mobile robot and asymptotic stability of the desired set.

## V. Controller

In this section, we propose a discontinuous asymptotic stabilizing state feedback control law for navigating the robot to the desired set $\Pi$. The procedure of designing the proposed control law consists of three steps: constructing a potential function $V$ on $X$ via the minimum projection method, calculating generalized gradients of the function $V$ and designing control law using generalized gradients.

### A. Construction of a potential function

The minimum projection method requires a mapping $\phi$. We consider the following surjection $\phi : \mathbb{R}^3 \to X$:

$$\phi(\tilde{q}) = \begin{bmatrix} \phi_x(\tilde{q}) \\ \phi_y(\tilde{q}) \\ \phi_\theta(\tilde{q}) \end{bmatrix} := \begin{bmatrix} r_{polar}(\tilde{q}) \cos \tilde{q}_\varphi - p_d \\ r_{polar}(\tilde{q}) \sin \tilde{q}_\varphi \\ \tilde{q}_\theta \end{bmatrix}, \tag{11}$$

$$r_{polar}(\tilde{q}) := \begin{cases} \tilde{q}_r + p_d & (\tilde{q}_r \geq 0) \\ \dfrac{2}{\pi}(p_d - r_d) \tan^{-1}\left(\dfrac{\pi \tilde{q}_r}{2(p_d - r_d)}\right) + p_d & (\tilde{q}_r < 0), \end{cases} \tag{12}$$

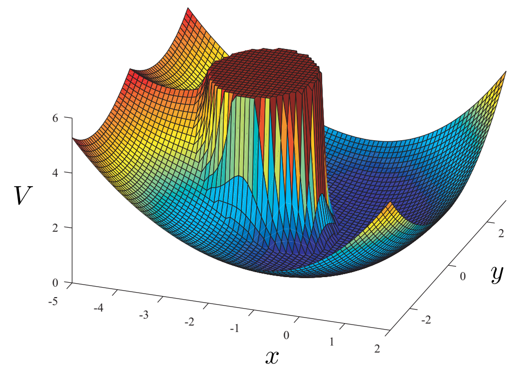


Fig. 1. Potential function : The minimum projection method

where $\tilde{q} = (\tilde{q}_r, \tilde{q}_\varphi, \tilde{q}_\theta) \in \tilde{X}$. Then, we can obtain a potential function $V$ via the minimum projection method as follows:

$$V(q) = \min_{\tilde{q} \in \phi^{-1}(q)} \tilde{V}(\tilde{q}) = \frac{1}{2}(\tilde{\varphi}_m(q)^2 + \tilde{r}_m(q)^2), \tag{13}$$

$$\tilde{V}(\tilde{q}) := \frac{1}{2}\tilde{q}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tilde{q}, \tag{14}$$

$$\tilde{r}_m(q) := \begin{cases} d(q) - p_d & (p_d \leq d(q)) \\ \dfrac{2}{\pi} \tan\left(\dfrac{\pi(d(p) - p_d)}{2(p_d - r_d)}\right) & (r_d < d(q) < p_d), \end{cases} \tag{15}$$

$$\tilde{\varphi}_m(q) := \arg(q_x + p_d + q_y i), \tag{16}$$

$$d(q) := \sqrt{(q_x + p_d)^2 + q_y^2}, \tag{17}$$

where $\arg : \mathbb{C} \to (-\pi, \pi]$ denots an argument of the complex number and $\arg(0) := 0$. Fig. 1 shows the potential function $V$, where $r_d = 1$ and $p_d = 2$. Note that the domain of Fig. 1 is $\mathbb{R}^2 \backslash D$ because $V$ does not depend on $\theta$.

### B. Calculation of generalized gradients

We can not define a gradient because potential function $V$ is not $C^1$ differentiable. Hence, we calculate one of the generalized gradients $[V_x, V_y, V_\theta]$ as follows.

$$[V_x, V_y, V_\theta] := -\left(\frac{\partial\tilde{V}}{\partial\tilde{q}}\left(\frac{\partial\phi}{\partial\tilde{q}}\right)^{-1}\right)(s(q)), \tag{18}$$

where $s : X \to \tilde{X}$ defined as follows:

$$s(q) := (\tilde{r}_m(q), \tilde{\varphi}_m(q), q_\theta) \tag{19}$$

**Remark 2** *$[V_x, V_y, V_\theta](q) = -\frac{\partial V}{\partial q}(q)$ if $V$ is differentiable on $q \in X$. Hence, we use the same notation of the generalized gradients as the notation of standard partial derivatives.*

### C. Controller that moves robot to direction of generalized gradients

We propose the following controller.

$$u_v(q) := k_1(V_x(q)\cos q_\theta + V_y(q)\sin q_\theta), \tag{20}$$

$$u_\omega(q) := \begin{cases} k_2 \arg((V_x(q)+V_y(q)i)\exp(-q_\theta i)) \\ \quad (-\pi/2 \le e(q) \le \pi/2) \\ k_2 \arg((V_x(q)+V_y(q)i)\exp(-q_\theta i)\exp(\pi i)) \\ \quad (-\pi < e(q) < -\pi/2, \pi/2 < e(q) \le \pi), \end{cases} \tag{21}$$

$$e(q) := \arg((V_x(q)+V_y(q)i)\exp(-q_\theta i)), \tag{22}$$

where $k_1, k_2 \in (0,\infty)$.

Then, we obtain the following theorem.

**Theorem 1** *When we apply the control law (20) and (21) to nonholonomic system (5),* $\Pi$ *is globally asymptotically stable in the sense of the Euler-solution.*

### D. Proof of Theorem 1

We suppose that $\sup_{i\in\mathbb{N}}(t_{i+1}-t_i)$ is sufficiently small for a given partition $\pi : i \mapsto t_i$. Due to difficulties in directly proving the theorem, we prepared the following 8 lemmas.

**Lemma 1** $(V_x(q), V_y(q)) = (0,0)$ *if and only if* $(q_x, q_y) = (0,0)$.

*Proof:* According to the definition of $(V_x(q), V_y(q))$,

$$\frac{\partial \tilde{V}}{\partial \tilde{q}}\left(\frac{\partial \phi}{\partial \tilde{q}}\right)^{-1} = [\tilde{q}_r, \tilde{q}_\varphi, 0]\begin{bmatrix} \frac{\partial \phi_x}{\partial \tilde{q}_r} & \frac{\partial \phi_y}{\partial \tilde{q}_r} & 0 \\ \frac{\partial \phi_x}{\partial \tilde{q}_\varphi} & \frac{\partial \phi_y}{\partial \tilde{q}_\varphi} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{23}$$

According to $|\partial\phi/\partial\tilde{q}| \ne 0$, $(\tilde{q}_r, \tilde{q}_\varphi) = (0,0)$ if and only if $(V_x(q), V_y(q)) = (0,0)$. According to the definition of $s$, $s$ is bijection and the image of $\{q | q_x = q_y = 0\}$ under $s$ is $\{\tilde{q} | \tilde{q}_r = \tilde{q}_\varphi = 0\}$. Hence, $(q_x, q_y) = (0,0)$ if and only if $(\tilde{q}_r, \tilde{q}_\varphi) = (0,0)$ and the lemma is proved. ■

**Lemma 2** *Assume the case that* $V_x(q)\cos q_\theta + V_y(q)\sin q_\theta = 0$ *and* $(q_x, q_y) \ne (0,0)$*, then* $-V_x(q)\sin q_\theta + V_y(q)\cos q_\theta \ne 0$.

*Proof:* (1) Consider the case of $\cos q_\theta = 0$. $\sin q_\theta \ne 0$ and $V_y(q) = 0$ by the assumption. $(V_x(q), V_y(q)) \ne (0,0)$ by Lemma 1. Therefore, $V_x(q) \ne 0$ and $-V_x(q)\sin q_\theta + V_y(q)\cos q_\theta \ne 0$.

(2) Consider the case of $\cos q_\theta \ne 0$ and $V_y(q) = 0$. $-V_x(q)\sin q_\theta + V_y(q)\cos q_\theta = \frac{V_y(q)}{\cos q_\theta}$ because $V_x(q) = -V_y(q)\frac{\sin\theta}{\cos\theta}$. $V_x(q) = 0$ by $V_x(q)\cos q_\theta + V_y(q)\sin q_\theta = 0$. This is a contradiction to $(V_x(q), V_y(q)) \ne (0,0)$.

(3) Consider the case of $\cos q_\theta \ne 0$ and $V_y(q) \ne 0$. According to $-V_x(q)\sin q_\theta + V_y(q)\cos q_\theta = \frac{V_y(q)}{\cos q_\theta}$, $-V_x(q)\sin q_\theta + V_y(q)\cos q_\theta \ne 0$. ■

**Lemma 3** $\psi_\theta(t_{i+1}) - \psi_\theta(t_i) \ne 0$ *when* $(\psi_x(t_i), \psi_y(t_i)) \ne (0,0)$ *and* $V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i) = 0$

*Proof:* Substitute $V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i) = 0$ into (20), we obtain $u_v(t_i) = 0$. According to $\dot{q}_x = u_v\cos q_\theta$ and $\dot{q}_y = u_v \sin q_\theta$, $\dot{q}_x(t_i) = 0$ and $\dot{q}_y(t_i) = 0$. Note that $(V_x(\psi(t_i)), V_y(\psi(t_i))) \ne (0,0)$ by Lemma 1.

$$\begin{aligned}\cos(e(\psi(t_i))) &= \cos(\psi_\theta(t_i) - \arg(V_x(\psi(t_i)) + V_y(\psi(t_i))i)) \\ &= \frac{V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i)}{\sqrt{V_x^2(\psi(t_i)) + V_y^2(\psi(t_i))}} \\ &= 0. \end{aligned} \tag{24}$$

Hence, $e(\psi(t_i)) \ne 0$ and $u_\omega(t_i) \ne 0$. According to $\dot{q}_\theta = u_\omega$, $\psi_\theta(t_{i+1}) - \psi_\theta(t_i) = (t_{i+1} - t_i)\dot{q}_\theta(t_i) \ne 0$ ■

**Lemma 4** $V_x(\psi(t_{i+1}))\cos\psi_\theta(t_{i+1}) + V_y(\psi(t_{i+1}))\sin\psi_\theta(t_{i+1}) \ne 0$ *when* $V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i) = 0$ *and* $(\psi_x(t_i)), \psi_y(t_i)) \ne (0,0)$.

*Proof:* According to Lemma 3, $\Delta\theta := \psi_\theta(t_{i+1}) - \psi_\theta(t_i) \ne 0$. By $\sin\Delta\theta \ne 0, \cos\Delta\theta \ne 0$ and Lemma 2,

$$\begin{aligned} &V_x(\psi(t_{i+1}))\cos\psi_\theta(t_{i+1}) + V_y(\psi(t_{i+1}))\sin\psi_\theta(t_{i+1}) \\ &= V_x(\psi(t_{i+1}))\cos\psi_\theta(t_{i+1}) + V_y(\psi(t_{i+1}))\sin\psi_\theta(t_{i+1}) \\ &- \sin\Delta\theta(-V_x(\psi(t_i))\sin\psi_\theta(t_i) + V_y(\psi(t_i))\cos\psi_\theta(t_i)) \\ &\ne 0. \end{aligned} \tag{25}$$

■

**Lemma 5** *When* $V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i) \ne 0$, $V(\psi(t_{i+1})) - V(\psi(t_i)) < 0$.

*Proof:* Since $\tilde{V}$ is $C^1$ function, following function $\tilde{H} : \mathbb{R}_{\ge 0} \times \mathbb{R}_{\ge 0} \to \mathbb{R}$ is well-defined:

$$\begin{aligned}\tilde{H}(t_{i+1}, t_i) :=& \tilde{V}(s(\psi(t_{i+1}))) - \tilde{V}(s(\psi(t_i))) \\ &- (t_{i+1} - t_i)\frac{d\tilde{V}}{dt}(s(\psi(t_i))). \end{aligned} \tag{26}$$

Note that $t_{i+1} - t_i \ne 0$ and the definition of derivation. Then,

$$\begin{aligned}\lim_{t_{i+1}\to t_i}\frac{\tilde{H}(t_{i+1}, t_i)}{t_{i+1} - t_i} &= \lim_{t_{i+1}\to t_i}\frac{\tilde{V}(t_{i+1}) - \tilde{V}(t_i)}{t_{i+1} - t_i} - \frac{d\tilde{V}}{dt}(t_i) \\ &= 0.\end{aligned}$$

$$\begin{aligned} &\frac{d\tilde{V}}{dt}(s(\psi(t_i))) \\ &= \left(\frac{d\tilde{V}}{d\tilde{q}}\left(\frac{\partial\phi}{\partial\tilde{q}}\right)^{-1}\right)(s(\psi(t_i)))g(\psi(t_i))u(\psi(t_i)) \\ &= -\begin{bmatrix} V_x(\psi(t_i)) & V_y(\psi(t_i)) & 0 \end{bmatrix} g(\psi(t_i))u(\psi(t_i)) \\ &= -k_1(V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i))^2. \end{aligned} \tag{27}$$

Note that $V(q) = \tilde{V}(s(q))$ and $V(q) = \min_{\tilde{q}\in\phi^{-1}(q)}\tilde{V}(\tilde{q})$.

$$\begin{aligned} &V(\psi(t_{i+1})) - V(\psi(t_i)) \\ &\le \tilde{V}(\psi(t_{i+1})) - \tilde{V}(\psi(t_i)) \\ &= (t_{i+1} - t_i)\left(\frac{d\tilde{V}}{dt}(s(\psi(t_i))) + \frac{\tilde{H}(t_{i+1}, t_i)}{t_{i+1} - t_i}\right) \\ &< 0. \end{aligned} \tag{28}$$

■

**Lemma 6** $\Pi$ *is stable.*

*Proof:*
Consider the case of $(\psi_x(t_i), \psi_y(t_i)) = (0,0)$. In this case, $\dot{q}_x = \dot{q}_y = 0$.
Consider the case of $(\psi_x(t_i), \psi_y(t_i)) \neq (0,0)$. According to Lemma 5, $V(\psi(t_{i+1})) - V(\psi(t_i)) < 0$ if $V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i) \neq 0$. According to Lemma 4, if $V_x(\psi(t_i))\cos\psi_\theta(t_i)+V_y(\psi(t_i))\sin\psi_\theta(t_i) = 0$, $V_x(\psi(t_{i+1}))\cos\psi_\theta(t_{i+1})+V_y(\psi(t_{i+1}))\sin\psi_\theta(t_{i+1}) \neq 0$. $V(\psi(t_{i+2})) - V(\psi(t_i)) < 0$.
Hence, every neighborhood $U$ of $\Pi$ contains a positively invariant set $\{q_l | q_l \in X, V(q_l) \leq \min_{q_{min}\in U} V(q_{min})\} \subset U$. ■

**Lemma 7** $\Lambda^+(q) \neq \emptyset$ *for every initial point* $q \in X$.

*Proof:* According to the compact desired set $\Pi$ is stable, Euler solution $\psi(q,t)$ stays in a compact level set $\{q_l | q_l \in X, V(q_l) \leq V(x_0)\} \subset U$. Consider Euler solution $\psi(t)$, a partition $\pi : \mathbb{N} \to \mathbb{R}_{\geq 0}; i \mapsto t_i$ and the sequence $\tilde{\pi} : \mathbb{N} \to X; i \mapsto q(t_i)$ on the compact level set, this sequence $\tilde{\pi}$ has Cauchy subsequence. There is a limit point of the sequence $\tilde{\pi}$, and the point belongs to $\Lambda^+(q)$ because every Cauchy sequence on a compact set has the limit point in the set. The lemma is proved. ■

**Lemma 8** $\Pi$ *is a global attractor.*

*Proof:* Assume that $\Pi$ is not a global attractor. By Lemma 7 and the definition of a global attractor, There exists an initial point $q \in X$ such that $\Lambda^+(q) \not\subset \Pi$; there exists $q_\omega \in X$ such that $q_\omega \in \Lambda^+(q)$ and $q_\omega \notin \Pi$.
According to Lemma 6, $\lim_{i\to\infty} V(\psi(t_i)) = V(q_\omega)$. $\lim_{i\to\infty} V(\psi(t_{i+1})) - V(\psi(t_i)) = 0$. Due to (27) and (28), the following equation holds:

$$\lim_{i\to\infty} V_x(\psi(t_i))\cos\psi_\theta(t_i)+V_y(\psi(t_i))\sin\psi_\theta(t_i)=0. \quad (29)$$

According to $\dot{q}_x = u_v\cos q_\theta$ and $\dot{q}_y = u_v \sin q_\theta$ and (20), $\lim_{i\to\infty}(\dot{q}_x(\psi(t_i)), \dot{q}_y(\psi(t_i))) = (0,0)$. Hence, $\lim_{i\to\infty}\dot{q}_\theta(\psi(t_i)) = 0$. $\lim_{i\to\infty}(V_x(\psi(t_i)), V_y(\psi(t_i))) \neq (0,0)$ by $q_\omega \notin \Pi$.

$$\begin{aligned} 0 &= \lim_{i\to\infty} V_x(\psi(t_i))\cos\psi_\theta(t_i) + V_y(\psi(t_i))\sin\psi_\theta(t_i) \\ &= \lim_{i\to\infty}\sqrt{V_x^2(\psi(t_i)) + V_y^2(\psi(t_i))} \\ &\quad \cos(\psi_\theta(t_i) - \arg(V_x(\psi(t_i)) + V_y(\psi(t_i))i)) \\ &= \lim_{i\to\infty}\cos(e(\psi(t_i))). \end{aligned}$$

Hence, $\lim_{i\to\infty} e(\psi(t_i)) \neq 0$ and $\lim_{i\to\infty} u_\omega(\psi(t_i)) \neq 0$. This is a contradiction to $\lim_{i\to\infty}\dot{q}_\theta(\psi(t_i)) = 0$ and $\dot{q}_\theta = u_\omega$. The lemma is proved. ■
Now, we ready to the proof of the Theorem 1.

*Proof:* [proof of Theorem 1.] According to Lemma 6 and Lemma 8, $\Pi$ is stable and an global attractor in the sense of the Euler-solution. ■

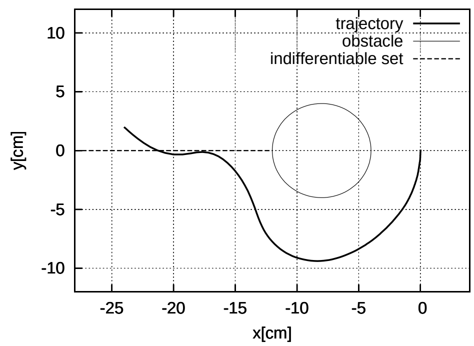


Fig. 2. Experimental result: Trajectory in the $\mathbb{R}^2\backslash D$ plane from $(-24, 2, -\pi/4)$

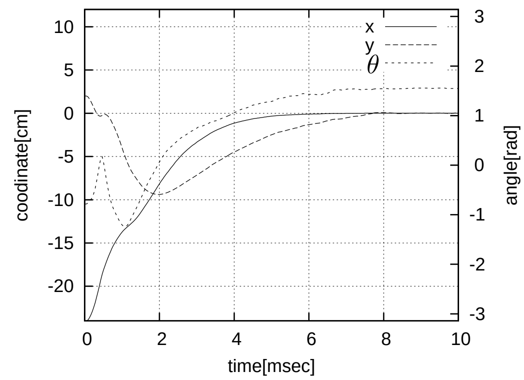


Fig. 3. Experimental result: Time variation of the state from $(-24, 2, -\pi/4)$

## VI. Experiment using mobile robot Khepera2

In this section, we confirm the effectiveness of the proposed method by an experiment using a two-wheeled mobile robot, Khepera 2.
Khepera2 is modeled by (5) and we can apply the control law (20) and (21) to the robot.
Figs. 2-3 show the experimental result with initial state$(-24, 2, -\pi/4)$[cm,cm,rad],$p_d = 8$[cm], $r_d = 4$[cm] , $k_1 = 10$ and $k_2 = 10$. We can confirm that Khepera2 avoids the obstacle and arrives at the desired set $\Pi$.
The robot may cross the set of indifferentiable points in the case where $\tilde{r}$ decreases much and $\tilde{\psi}$ increases little. This means that the indifferentiable set does not imply decision-making. The reason is that nonholonomic constrained robots cannot move in direction $(V_x, V_y)$.

## VII. Compare with the artificial potential method

The minimum projection method and the artificial potential method [2] are the real-time designing potential function methods. The diffirence is as follows. The formar method garantee global stability. The later method almost garantee stability except some points called suddle points.
In this section, we try to use controller (20) and (21) with the potential designed by the artificial potential method.

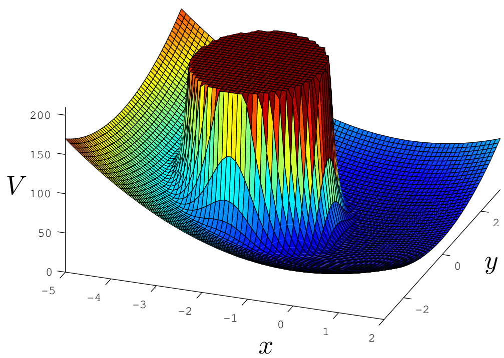


Fig. 4. Potential function: The artificial potential method

### A. artificial potential method

In the artificial potential method, the potential function $U : X \to \mathbb{R}$ is defined as follows:

$$U(q) := U_{xd}(q) + U_o(q), \tag{30}$$

$$U_{xd}(q) := \frac{1}{2}k_p(x^2 + y^2), \tag{31}$$

$$U_o(q) := \begin{cases} \frac{1}{2}\eta(\frac{1}{\rho(p)} - \frac{1}{\rho_0})^2 & (\rho(q) \le \rho_0) \\ 0 & (\rho(q) > \rho_0), \end{cases} \tag{32}$$

where $U_{xd} : X \to \mathbb{R}$ is an attracting force from the desired set, $U_o : X \to \mathbb{R}$ is a repulsive force from the obstacle, $\rho(p)$ is the distance between current position $(x, y)$ and nearest obstacle, and $k_p, \rho_0, \eta \in (0, \infty)$.

In this paper's situation, we can set up $\rho(q) = d(q) - r_d$. Fig. **??**fig:clfapm shows the potential function using the artifisial potential method with $k_p = 10, \rho_0 = 1, \eta = 200$ and $p_d = 2$. Fig. 4 has the saddle point in the opposite side of the origin. By contrast, Fig. 1 has a undifferential region. This is the difference between Fig. 4 and Fig. 1.

### B. experiment using artificial potential method

Instead of (18), we set up the gradient $[V_x, V_y, V_\theta]$ as follows:

$$[V_x, V_y, V_\theta] := -\frac{\partial U}{\partial p}. \tag{33}$$

Figures. 5-6 show the experimental result with initial state $(-24, 2, -\pi/4)$[cm,cm,rad], $p_d = 8$[cm], $r_d = 4$[cm], $k_1 = 10$ and $k_2 = 5$.

### C. discussion

The experiment result shows that the saddle point $(x, y) = (-15, 0)$ attracts the mobile robot. This means that the single saddle point makes large stable region. Hence, the artificial potential method can not use for controlling the mobile robot. The proposed method avoids this probrem. Moreover the method acchives real-time and global control.

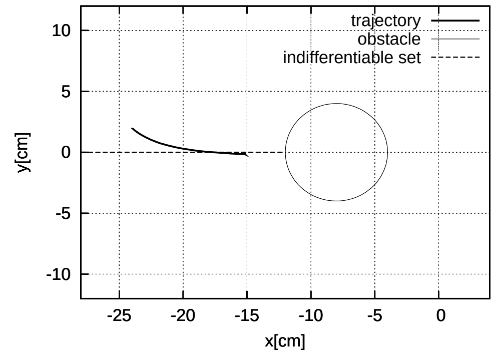


Fig. 5. Experiment result: Trajectory in the $\mathbb{R}^2 \backslash D$ plane by the artificial potential method from $(-24, 2, -\pi/4)$

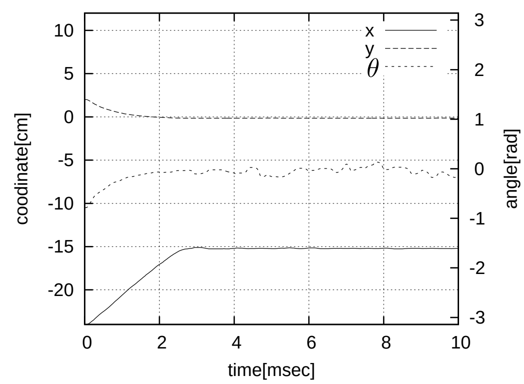


Fig. 6. Experiment result: Time variation of the state from $(-24, 2, -\pi/4)$

## VIII. CONCLUSION

In this paper, we proposed a discontinuous asymptotic stabilizing state feedback control law for a real-time obstacle avoidance problem via the minimum projection method. We proved that the desired set is asymptotic stable when we apply the proposed method to the robot. The method achieves real-time and global control. Moreover, we confirmed this effectiveness by the experiments.

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