A constraint in single-feedback Sigma Delta Force-Feedback Loops with a discrete-time loop filter

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***Abstract*— In this tutorial paper we review Sigma Delta force feedback for the read-out of Micro-Electro-Mechanical (MEMS) inertial sensors with a discrete time loop filter. First we focus on the single feedback structure (with only mechanical feedback). It is shown that in this situation the mechanical transfer function introduces a zero in the overall loop gain. This gives rise to a constraint in the realizable NTF. In theory this can be overcome by adding a pole to the controller. An alternative solution is to add an extra electrical feedback branch. The latter solution is considered more beneficial in terms of power consumption and chip area.**

I. Introduction

Micro Electro Mechanical Systems (MEMS) consist of a (micro)mechanical element combined with interfacing electronic circuits. Such MEMS allow to built inertial sensors (measuring acceleration or rotation rate) [1]–[10]. Here the mechanical element consists of a mass with a spring. In the presence of an inertial force F the mass will move, leading to a displacement x. Taking also damping into account, this can be described by the well known second order mass-damper-spring transfer function $T_{mech}(s)$:

$$T_{mech}(s) = \frac{x(s)}{F(s)} = \frac{1/k}{1 + \zeta\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2} \quad (1)$$

This displacement x can then be measured electronically, e.g. by a capacitive read-out amplifier, which usually also performs a sampling operation. This gives the read-out voltage V_{read}. In the simplest case, this read-out voltage is the sensor's output signal. In practice, this read-out voltage can then be filtered and convertered to the digital domain by an additional A/D converter (see Figure 1).

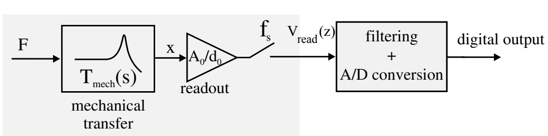

Fig. 1. Diagram of a MEMS inertial sensor without feedback.

Unfortunately, there are various problems with this straightforward open loop topology. E.g. it is clear that the relationship between the read-out voltage and the actual displacement must be accurately known and preferrably linear. Also the paramaters of equation (1) must be accurately known. In practice this is difficult to achieve since they change with temperature and due to aging.

II. Sigma Delta Force feedback

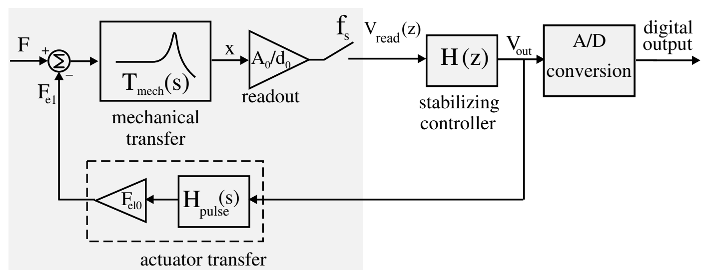

Fig. 2. Diagram of an analog MEMS force-feedback loop.

To avoid the problems associated with the straightforward open loop approach of figure 1, force feedback can be used. The concept is shown in figure 2. Here the mechanical sensor is adapted such that there is also a way to apply an electrically controlled force to the mass: i.e. an actuator is added. In most cases capacitive actuation is used. Then this actuation is used to apply force feedback. Here the actuator is driven by the analog output voltage of the sensor V_{out}. By the operation of the control loop, this output voltage V_{out} will be adapted untill the electrical force F_{el} applied through the actuator is equal to the input inertial force F. This way V_{out} directly provided information on the inertial force F and can be used as the sensor's output. If needed the analog output voltage can be converted to the digital domain by an additional A/D converter.

This topology solves the problems associated with the open loop structure of figure 1. However a new problem is introduced: the relationship between the voltage V_{out} and the actuator force F_{el} should be perfectly linear, known and stable over time and temperature. However, it is well known that the relationship between the actuation force and the actuation voltage is quadratic. Some linearization techniques exist, but their accuracy is limited by mismatch of the actuation devices. This way, this is not unfeasable but not very convenient.

For this reason the force feedback loop of figure 2 can be converted into a Sigma Delta force feedback loop shown in figure 3, top ([1], [3]–[10]). Here, the sensor is also embedded in a feedback loop, but now a 1-bit quantizer (comparator) is added to the structure. This structure has several advantages over the complete analog force feedback

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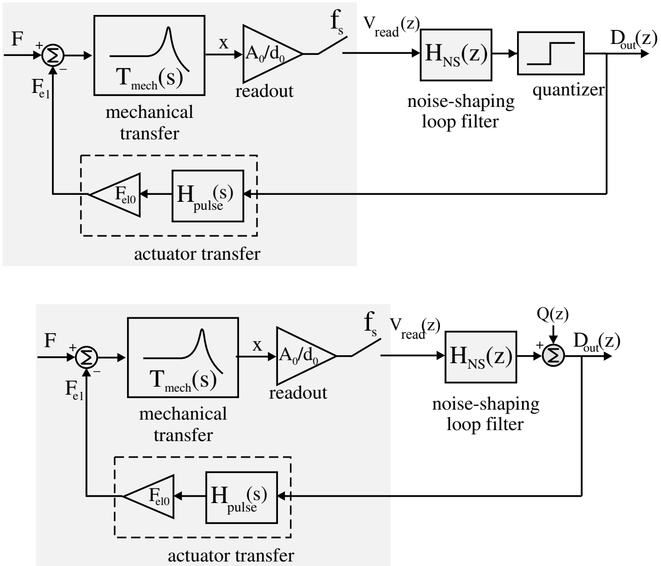

Fig. 3. Diagram of a MEMS Sigma Delta force-feedback loop (top) and its pseudo linearmodel (bottom).

loop of figure 2. First, the actuation is inherently linear because now only 2 actuation levels are used. Second, the output signal of the comparator is already digital. This obsoletes the use of an external A/D converter. The structure also has a few drawbacks, the main of which is the hard non-linear action of the quantizer in the loop. We will explain how we can deal with this later on.

We will now study the structure of figure 3 (top) in more detail. Referring to the figure we notice the mechanical transfer $T_{mech}(s)$, described by eq. (1). Its output is the displacement x. The read-out is assumed to be perfectly linear with a normalized gain A_0/d_0. The normalization distance d_0 is not relevant for this discussion, but refers to the gap of the read-out capacitors. In practice the read-out amplifier also performs a sampling operation. Then the read-out voltage V_{read} is applied to an additional loopfilter filter H_{NS} which we will call the the noise shaping filter. Since the read-out voltage is sampled, this is a discrete time filter. The 1-bit quantizer determines the digital output squence D_{out} based on the loop-filter output. This digital output signal can take two values (± 1) and drives the actuator. This way, the output signal of the actuator (which is the actuation force F_{el}) can also only take two values $\pm F_{el0}$. Since the output signal D_{out} is a discrete time signal, and the mechanical force F_{el} is a continuous time signal, we should also consider the transfer function of the associated pulse $H_{pulse}(s)$. In many cases this pulse will be a simple zero-order-hold pulse, but also return-to-zero pulses have been used. However, in nearly all cases the pulse will be rectangular:

$$H_{pulse}(s) = \frac{e^{-s\tau_1} - e^{-s\tau_2}}{s}, \qquad 0 \leq \tau_1 < \tau_2 \leq \frac{1}{f_S}. \tag{2}$$

Here τ_1 and τ_2 correspond to the start and stop instance of the pulse and f_S to the sampling frequency. Note that there is only one feedback path from the output (D_{out}) to the input. This is why such a structure is called a single-feedback structure. A common way to analyse this structure introduces the so-called quantization error Q which is defined as the difference between the input of the quantizer and its output. This results in the system shown in figure 3, bottom.

When looking at the diagram of fig. 3, we notice that the signal D_{out} is a discrete-time signal (sequence), and also the read-out voltage V_{read} is a discrete time signal. As is common we will analyse these signals in the Z-domain, i.e. by considering the Z-transforms $D_{out}(z)$ and V_{read}(z). By inspection it is clear that there exists a linear and time invariant relationship between D_{out} and V_{read}. Thus, there exists a Z-domain transfer function $H_{mech}(z)$ such that:

$$V_{read}(z) = H_{mech}(z) D_{out}(z) \tag{3}$$

It is clear that this transfer function $H_{mech}(z)$ is determined by the the actuator transfer function $F_{el0}H_{pulse}(s)$ and the mechanical transfer function $T_{mech}(s)$. This dependency is known as the impulse invariant transform. The most straightforward way to calculate it is by taking the inverse Laplace transform, then sampling the result and, then taking the Z-tranform:

$$H_{mech}(z) = \mathcal{Z}\left[\mathcal{L}^{-1}\left(F_{el0}H_{pulse}(s)T_{mech}(s)\right)\Big|_{t=nT}\right] \tag{4}$$

In practice it is more convenient to calculate the impulse invariant transform by using the following equation:

$$H_{mech}(z) = \sum_k \text{residus}\left[\frac{e^{\frac{\chi}{f_S}}}{z - e^{\frac{\chi}{f_S}}} \cdot T_{mech}(\chi)F_{el0}H_{pulse}(\chi)\right] \quad \text{in } \chi = s_k$$

This equation is easy to evaluate and in fact is incorporated in popular tools such as Octave and Matlab.

Combining this with equation (1) and (2), it can be shown that $H_{mech}(z)$ takes the following form [10]:

$$H_{mech}(z) = A\frac{(z - z_z)}{(z - z_m)(z - z_m^*)} \tag{5}$$

Here, A is a constant, z_m and z_m^* correspond to the mechanical poles which are related to the poles s_m, s_m^* of the mechanical system of equation (1), as $z_m = e^{\frac{s_m}{f_S}}$ and $z_m^* = e^{\frac{s_m^*}{f_S}}$. Additionally it can be shown that there always is a real zero z_z as well.

Now, the pseudo-linear system of fig. 3(bottom) can be solved. By inspection it is clear that the digital output D_{out} can be written as a sum of a contribution proportional to the inertial force F and a contribution proportional to the quantization error Q. Since the goal of this system is to measure the inertial force F, the term proportional to F is the desired output, while the term proportional to Q is an undesired quantization error contribution. Let us therefore focus on the term proportional to Q, which is readily obtained by setting the input force equal to zero and calculating the output signal:

$$D_{out}(z)|_{F=0} = Q(z)\frac{1}{1 + H_{NS}H_{mech}(z)} = NTF(z)Q(z) \tag{6}$$

In the literature on electrical sigma delta modulators, the transfer function of the quantization error Q to the digital output D_{out} is called the quantization noise transfer function NTF. Since the quantization error corresponds to an undesired contribution, it is important that the NTF suppresses this contribution as much as possible in the frequency band that is relevant for the sensor operation. This is achieved by designing the overall control loop such that the magnitude of the NTF is as small as possible in the frequency band of interest.

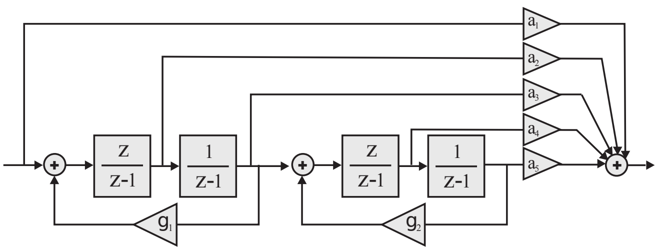

Fig. 4. Potential realisation of fourth-order noise shaping loop filter.

In practice this is done by adding poles in the signal band to the noise shaping filter H_{NS}. To illustrate this, let us assume for the moment that we are interested in the low-frequency signal band. Then a practical loop filter could look like figure 4. It consists of a cascade of integrators that have an infinite gain at DC. The g_i coefficients are very small coefficients. Their purpose is to shift the poles of H_{NS} slightly away from DC and to optimally spread them over the signal band [11], [13]. However for a first analysis they can be assumed to be zero. The a_i coefficients are needed to set the zeros of H_{NS} and these are needed to control the poles of the NTF, see eq. (6). The number of integrators in a filter as shown in figure 4 is called the order of the loop filter. In practice we will have better suppression of the quantization noise for a higher loop filter order.

In the context of purely electrical $\Sigma\Delta$ modulators there is a consensus that the best approach for the design of such a control loop consists of 2 steps [11]–[13]. First a suitable choice is made for the NTF, and then in a second step the loop filters are determined which provide the desired NTF. With this in mind, a promising way to design a force feedback loop would be to adopt this same approach, i.e. first determine a good choice of the NTF and then determine the loop filter. However, here the mechanical system is in the loop. This mechanical system is governed by equation (1) and (5), and hence its the form of its transfer function is cannot be controlled by the designer. Only the electrical filter H_{NS}, is fully controlled by the designer. Referring to figure 4, this way the design of a Sigma Delta force feedback loop boils down to finding suitable values for the a_i coefficients.

III. Constraint on the NTF

A. Basic constraint

Unfortunately it is not possible to find values for the a_i coefficients such that an arbitrary noise transfer function is realized. The reason for this is that any practical NTF with a loop filter such as figure 4, will exhibit a constraint. This is readily observed by evaluating the NTF at the zero of the discrete time equivalent of the mechanical transfer: i.e. for $z = z_z$.

$$NTF(z)|_{z=z_z} = \frac{1}{1 + H_{NS}H_{mech}(z)}|_{z=z_z} = 1 \qquad (7)$$

In fact the above equation is only valid, if we assume that H_{NS} evaluated for $z = z_z$ is finite. However by simple inspection it is clear that a loop filter such as figure 4 indeed does not have a pole at $z = z_z$. As a result it is impossible to find a_i coefficients that allow optimal noise shaping.

B. Placing a loop filter pole at $z = z_z$

In practical $\Sigma\Delta$ control loops, only non-zero poles are added to the loop filter if this also suppresses the quantization noise. The reason for this is that increasing the order of the loop filter adds to the power and the complexity of the loop filter. Note that this is an analog filter and thus prone to all the difficulties of analog design. However, in theory it is thinkable that the electrical filter H_{NS} can have a pole at exactly the same location as z_z. Consider e.g. the case where the damping of the mechanical filter is negligible, this corresponds to $Q = \infty$ in eq. (1). Assume now a zero-order-hold feedback pulse (see eq. (2)). It is not difficult to show that in this situation the zero will be located at $z_z = -1$. This is still at the edge of the unit circle. In other cases (i.e. nonzero damping and/or pulse durations that are shorter than 1 clock cycle), z_z can be either within or outside the unit circle. This way it is thinkable to add an electrical filter section which cancels the mechanical zero. This situation is shown in figure 5.

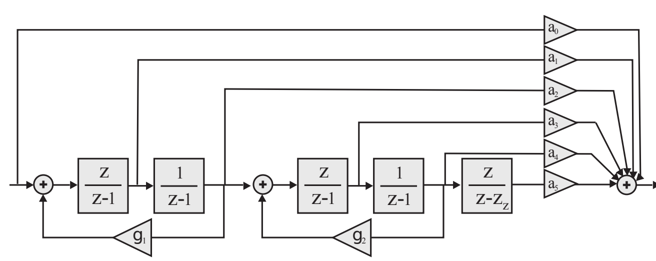

Fig. 5. Potential realisation of fourth-order noise shaping loop filter with an extra pole to cancel the mechanical zero.

It is obvious that such a force feedback loop does not have the constraint of eq. (7). Furthermore, it is easy to show that the associated loop filter has sufficient independent degrees of freedom (the 6 a_i coefficients) to realize arbitrary NTF pole positions. However, the structure has several drawbacks which apparently make it impractical. First, the pole is realized by an electrical circuit. But this pole should be matched to a zero that originates from the mechanical block. As such, in practice the matching between the pole and zero will not be very good. As a result, the cancellation will be imperfect and a doublet will occur in the overall loop gain. If this doublet is outside the unit circle, special care is needed.

Next, it is important to note that the incorporation of the additional pole at $z = z_z$, will require an additional operational amplifier and its associated silicon area and power consumption. Moreover, this area and power is spent without an increase of the order of the noise shaping (and its associated improved accuracy performance).

Still this approach should be feasable, although no actually working design incorporating this technique has been reported yet. This is probably due to the reasons explained above. Instead most designs rather implement an FIR compensation by adding a filter $H_{comp} = 1 - az^{-1}$ in cascade with the noise shaping filter. This can be implemented in a passive way, without additional power consumption [1], [4]–[6]. However, such an FIR-type compensation does not remove the mechanical zero z_z and hence can not resolve the constraintness of the architecture. In practice this leads to sub-optimal noise shaping [10].

C. Adding an extra feedback branch

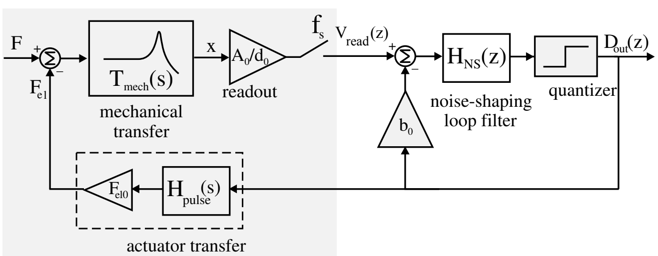

Fig. 6. Potential realisation of fourth-order noise shaping loop filter with an extra pole to cancel the mechanical zero.

A more attractive solution is shown in figure 6. Here an additional feedback path is added bypassing the mechanical path. The noise shaping loop filter H_{NS} is of the form shown in figure 4. Now, the noise transfer function will equal:

$$NTF(z) = \frac{1}{1 + H_{NS}H_{mech}(z) + b_0 H_{NS}}$$

It is clear that this NTF does not exhibit the constraint of equation (7). By simply writing down the transfer function, it is also easy to show that the associated loop filter has sufficient independent degrees of freedom (the 6 a_i coefficients) to realize arbitrary noise transfer functions.

Again, it can be shown by simple enumeration that the loop filter has sufficient degrees of freedom (the a_i and b_0 coefficients) to realize arbitrary noise transfer functions.

It is important to note that the addition of the electrical feedback branch does not require an additional amplifier. As such it has a negligible impact on the power consumption and chip area of the analog integrated circuit. This way this solution is preferred over the solution with the additional pole of figure 5, and was also used in [7], [9].

Still, also this solution has the disadvantage that it is sensitive to mismatch between coefficients occuring in the electrical path (e.g. the b_0 coefficients) and parameters of the mechanical structure (the actuator gain F_{el}, the mass m and the spring constant k). However it was found in practice that these matching conditions can be met without excessive trimming and tuning [7], [9].

IV. Conclusion

We have reviewed Sigma Delta Force feedback and explained why this is an interesting approach for the read-out of Micro-Electro-Mechanical (MEMS) inertial sensors. Next we have examined the single feedback structure and shown that this structure is prone to a constraint in the realizable NTF. This constraint originates from the zero in the Z-domain transfer function of the mechanical sensor.

We have shown that there are two ways to overcome this problem. The first solution is to add a pole in the electrical controller to cancel this zero. The second solution is to add an additional electrical feedback path. The former solution is the authors' preferred choice because it apparently allows a simpler circuit implementation.

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