LQ parametrization of Robust Stabilizing Static Output Feedback Controllers for 2D Continuous Roesser Models

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Abstract— **The paper considers robust control of 2D linear systems described by the Roesser model, where information propagation in each the independent directions is a function of a continuous variable, and an affine parallelotopic type model for uncertainty is assumed. Sufficient optimality conditions for the existence of an LQ state feedback controller are developed in the absence of model uncertainty and then used to characterize a set of stabilizing static output feedback controllers for this system in the presence of parameter uncertainties, resulting in non-convex conditions parameterized by the weighting matrices of a quadratic cost function. Replacing these conditions by convex approximations leads to an algorithm for computing the stabilizing gain matrix of the controller. The algorithm is non-iterative and uses computationally efficient SDP solvers. A numerical example is given to demonstrate the applicability and effectiveness of the algorithm.**

I. INTRODUCTION

Multidimensional systems propagate information in $n > 1$ independent directions but in this paper attention is restricted to the 2D case where the dynamics evolve over the right-upper quadrant of the associated plane. The study of 2D systems is motivated by many applications in, for example, image and signal processing and also by systems theoretic questions that cannot be solved by direct extension of standard, or 1D, theory. In terms of models for the dynamics, there is a much wider variety of signals possible where, for example, information propagation could be functions of discrete variables in both directions, of continuous variables in both directions, or a discrete variable in one direction and continuous in the other.

Consider the case when information propagation in both directions is a function of a discrete variable, then there are two extensively studied state-space models [1], [2]. The Roesser model defines a state vector for each direction of information propagation whereas the Fornasini-Marchesini model uses a single state vector. Repetitive processes [3] also have a 2D systems structure but information propagation in one of the two directions only occurs over a finite duration. In control systems terms, repetitive processes do provide physical applications, such as iterative learning control, where a 2D systems approach is required and this area has recently seen experimental verification studies [4].

Stability analysis and robust controller design for uncertain 2D discrete linear systems has received much attention. For example, using eigenvalue sensitivity, results on robust stability for uncertain 2D linear systems are given in [5], while in [6] a frequency domain and Lyapunov mapping approach respectively, were proposed for the robust stability problem. An LMI approach to robust stabilization has also been extensively studied, e.g. [7], [8]. The vast majority of the results currently available on control related analysis of 2D linear systems is for the discrete case or, for repetitive processes, continuous in one direction and discrete in the other. This paper considers 2D systems with continuous dynamics in both directions of information propagation described by a Roesser state-space model for which stability analysis in the presence of parameter uncertainty in the matrices of the state-space model is considered in [9], where the uncertainty is assumed to be norm-bounded.

This paper considers the same continuous Roesser state-space model as [9]–[11] but with an affine parallelotopic type model of uncertainties. The first set of results develop give sufficient conditions for optimality of an LQ state feedback controller and the conditions then used to describe a set of stabilizing static output feedback controllers. This results in non-convex conditions, parameterized by the weighting matrices of a quadratic cost function and application of a convexifying approximation technique leads to an LMI-based algorithm for computing the stabilizing controller gain matrix. The algorithm can be implemented without the need for iterations and uses computationally efficient SDP solvers. Finally, an illustrative numerical example is given.

Throughout this paper the notation $M > 0$ (respectively) $M < 0$ is used to denote a symmetric positive-definite (respectively negative-definite) matrix. Also $M \geq 0$ (respectively $M \leq 0$) is used to denote a symmetric positive (respectively negative) semi-definite matrix.

II. PROBLEM FORMULATION AND PRELIMINARIES

The systems considered in this paper are, in the absence of uncertainty, described by the 2D Roesser state-space model

$$\begin{aligned} \begin{bmatrix} \frac{\partial}{\partial t_1} h(t_1,t_2) \\ \frac{\partial}{\partial t_2} v(t_1,t_2) \end{bmatrix} &= A \begin{bmatrix} h(t_1,t_2) \\ v(t_1,t_2) \end{bmatrix} \\ &+ Bu(t_1,t_2), \\ z(t_1,t_2) &= C \begin{bmatrix} h(t_1,t_2) \\ v(t_1,t_2) \end{bmatrix}. \end{aligned} \qquad (1)$$

This work is supported in part by Russian Foundation for Basic Research under grants 08-01-97036, 10-08-00843 and also by the Polish Ministry of Science and Higher Education under the project N N514 293235

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ISBN 978-963-311-370-7

where $h \in \mathbb{R}^{n_h}$ and $v \in \mathbb{R}^{n_v}$ are the horizontal and vertical state vectors respectively, and $u \in \mathbb{R}^{n_u}$ and $z \in \mathbb{R}^{n_z}$ are the input and output vectors respectively. Now, compatibly partition the state matrix A in (1)

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

and introduce the characteristic polynomial

$$C(s_1, s_2) = \det \begin{bmatrix} s_1 I - A_{11} & -A_{12} \\ -A_{21} & s_2 I - A_{22} \end{bmatrix}, \quad (2)$$

and the following result gives the necessary and sufficient condition for asymptotic stability of the systems considered in this work.

Lemma 1: [10], [11] The continuous Roesser model of (1) is asymptotically stable if and only if $C(s_1, s_2)$ has no roots, or zeros, in the closed right-half of the biplane including the points at infinity, that is

$$C(s_1, s_2) \neq 0 \ \text{ if } \ (s_1, s_2) \in \bar{\bar{D}}^2, \quad (3)$$

where

$$\bar{\bar{D}}^2 = \{(s_1, s_2) : \text{Re } s_1 \geq 0, \ \text{Re } s_2 \geq 0, \ | s_1 | \leq \infty, | s_2 | \leq \infty\}.$$

The following result provides a sufficient but not necessary stability condition in terms of an LMI, which is computationally tractable relative to the condition of Lemma 1.

Lemma 2: [9], [12] The system (1) is asymptotically stable if there exist matrices $P_1 > 0$ and $P_2 > 0$ satisfying the following LMI

$$A^T P + PA < 0, \quad (4)$$

where $P = \text{diag}[P_1 \ P_2]$.

If there is uncertainty is associated with the system dynamics, one way of modeling this for analysis is to use an affine parallelotopic type model of the form

$$\begin{aligned} \begin{bmatrix} \frac{\partial}{\partial t_1} h(t_1, t_2) \\ \frac{\partial}{\partial t_2} v(t_1, t_2) \end{bmatrix} &= A(\delta) \begin{bmatrix} h(t_1, t_2) \\ v(t_1, t_2) \end{bmatrix} + B(\delta) u(t_1, t_2), \\ z(t_1, t_2) &= C \begin{bmatrix} h(t_1, t_2) \\ v(t_1, t_2) \end{bmatrix}, \end{aligned} \quad (5)$$

with variations around a central nominal model defined by matrices (A, B) along the axes (A_i, B_i)

$$A(\delta) = A + \sum_{i=1}^{N} \delta_i A_i \quad , \quad B(\delta) = B + \sum_{i=1}^{N} \delta_i B_i \quad . \quad (6)$$

Also δ_i is assumed to be bounded in an interval including zero

$$\underline{\delta}_i \leq \delta_i \leq \overline{\delta}_i \quad : \quad \underline{\delta}_i \leq 0 \quad , \quad \overline{\delta}_i \geq 0 \ . \quad (7)$$

The set of uncertainties is denoted $\mathbf{\Delta}$ and the finite set of extremal values, or vertices, defined by

$$\mathbf{\Delta}_v = \left\{ \ \delta = \begin{pmatrix} \delta_1 & \dots & \delta_N \end{pmatrix} : \ \delta_i \in \{\underline{\delta}_i, \overline{\delta}_i\} \ \right\} \quad (8)$$

Consider application of the static output feedback control law

$$u(t_1, t_2) = -F z(t_1, t_2). \quad (9)$$

to the model with uncertainty. The problem considered in this paper is to describe in a constructive form the set of controllers in (9) that guarantee asymptotic stability of (5) for all uncertainties satisfying (7).

III. LQ OPTIMAL CONTROL FOR 2D ROESSER MODELS

In 1D linear systems theory the LQR setting gives a constructive description of the set of state feedback controllers in terms of its weighting matrices and hence the solution to static output feedback problem. In this section we extend this approach to the 2D Roesser state-space model introduced in the previous section.

Write (5) as

$$\frac{\partial h(t_1, t_2)}{\partial t_1} = A_{11} h(t_1, t_2) + A_{12} v(t_1, t_2) + B_1 u(t_1, t_2), \quad (10)$$

$$\frac{\partial v(t_1, t_2)}{\partial t_2} = A_{21} h(t_1, t_2) + A_{22} v(t_1, t_2) + B_2 u(t_1, t_2). \quad (11)$$

and suppose that boundary state vectors $h(0, t_2)$ and $v(t_1, 0)$ satisfy

$$\lim_{T \to \infty} T^{-1} \int_0^T | h(0, t_2) |^2 \, dt_2 < \infty,$$

$$\lim_{T \to \infty} T^{-1} \int_0^T | v(t_1, 0) |^2 \, dt_1 < \infty.$$

Also define for continuous functions $f(h)$ on $\mathbb{R}^{n_h}$ and $g(v)$ on $\mathbb{R}^{n_h}$ the differential operators

$$\begin{aligned} \mathcal{L}_{vu} f(h) &= \frac{\partial f}{\partial h}(A_{11} h + A_{12} v + B_1 u) \\ \mathcal{L}_{hu} g(v) &= \frac{\partial g}{\partial v}(A_{21} h + A_{22} v + B_2 u) \end{aligned} \quad (12)$$

Then the control $u(t_1, t_2)$ in (11) is admissible if it has the form $u(t_1, t_2) = \varphi(h(t_1, t_2), v(t_1, t_2))$, where $\varphi(h, u)$ is continuous function on $\mathbb{R}^{n_h} \times \mathbb{R}^{n_v}$ such that

$$\varphi(h, v) \leq \kappa(1 + | h | + | v |), \ \kappa > 0, \quad (13)$$

Also the solution of (10) with $u = \varphi(h, u)$ satisfies

$$\lim_{T \to \infty} T^{-1} \int_0^T \int_0^T (| h(t_1, t_2) |^2 + | v(t_1, t_2) |^2) dt_1 dt_2 < \infty. \quad (14)$$

and in this case we write $u \in \Phi_a$.

For given $Q > 0$ and $R > 0$, introduce

$$L(h, v, u) = [h^T \ v^T] Q [h^T \ v^T]^T + u^T R u$$

and the cost function

$$J(u) = \lim_{T \to \infty} T^{-1} \int_0^T \int_0^T [L(h(t_1, t_2), v(t_1, t_2), u(t_1, t_2)] dt_1 dt_2$$

for (10). Then the admissible control $u = \varphi^0(h, v)$ is optimal if

$$J(\varphi^0(h, v)) \leq J(\varphi(h, v)) \ \ \varphi \in \Phi_a.$$

and the following result gives sufficient conditions for optimality.

Theorem 1: Suppose that there exists a function $\varphi^0(h, v) \in \Phi_a$ and real-valued functions $V_1(h)$ and $V_2(v)$ on $\mathbb{R}^{n_h}$ and on $\mathbb{R}^{n_v}$ with the following properties

(i) the functions $V_1(h), \frac{\partial V_1(h)}{\partial h}, V_2(v), \frac{\partial V_2(v)}{\partial v}$ are continuous on $\mathbb{R}^{n_h}$ and on $\mathbb{R}^{n_v}$ respectively, $V_1(0) = 0, \ V_2(0) = 0$,

(ii)

$$\mid V_1(h) \mid + \mid x \mid \frac{\partial V_1(h)}{\partial h} \mid + \mid V_2(v) \mid + \mid v \mid \frac{\partial V_2(v)}{\partial v} \mid \leq k(\mid h \mid^2 + \mid v \mid^2)$$

for some constant k, and

(iii)

$$\mathcal{L}_{v\varphi^0} V_1(h) + \mathcal{L}_{h\varphi^0} V_2(v) + L(h, v, \varphi^0(h, v)) = 0; \tag{15}$$

$$\mathcal{L}_{vu} V_1(h) + \mathcal{L}_{hu} V_2(v) + L(h, v, u)) \geq 0 \tag{16}$$

for all $(h, v, u) \in \mathbb{R}^{n_h} \times \mathbb{R}^{n_v} \times \mathbb{R}^{n_u}$.

Then $\varphi^0(h, v)$ is the optimal control.

Proof: Introduce

$$W = \lim_{T\to\infty} T^{-1} \left(\int_0^T V_1(h(0, t_2))dt_2 + \int_0^T V_2(v(t_1, 0))dt_2 \right).$$

and integrate this function to obtain

$$W = \lim_{T\to\infty} T^{-1} \int_0^T \int_0^T [L(h_{\varphi^0}(t_1, t_2), v_{\varphi^0}(t_1, t_2), \varphi^0(h_{\varphi^0}(t_1, t_2), v_{\varphi^0}(t_1, t_2)]dt_1 dt_2, \tag{17}$$

where the subscript φ^0 denotes the solution to (10) with $u = \varphi^0(h, u)$. Similarly, integrating (16) for some admissible $u = \varphi(h, v)$ gives

$$W \leq \lim_{T\to\infty} T^{-1} \int_0^T \int_0^T [L(h_{\varphi}(t_1, t_2), v_{\varphi}(t_1, t_2), \varphi(h_{\varphi}(t_1, t_2), v_{\varphi}(t_1, t_2)]dt_1 dt_2, \tag{18}$$

and it follows from (17) and (18) that

$$W = J(\varphi^0(h, v)) \leq J(\varphi(h, v)), \ \ \varphi, \ \varphi^0 \in \Phi_a.$$

Hence φ^0 is the optimal control. ■

To obtain a Bellman function interpretation of this last result, first note that (17) and (18) can be written as

$$\min_{u\in\mathbb{R}^{n_u}} \{\mathcal{L}_{vu} V_1(h) + \mathcal{L}_{hu} V_2(v) + L(h, v, u)\} = 0. \tag{19}$$

and the minimization in (19) gives

$$u = \varphi^0(u, v) = -R^{-1}[B_1^T \frac{\partial V_1}{\partial h} + B_2^T \frac{\partial V_2}{\partial v}] \tag{20}$$

Setting $V_1(h) = h^T P_1 h$ and $V_2(h) = v^T P_2 v$ now gives

$$u = -R^{-1} B^T P x, \tag{21}$$

where $x = [h^T \ v^T]^T$ and $P = \text{diag}[P_1 \ P_2] > 0$ solves the algebraic Riccati equation for some matrix $R > 0$

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \tag{22}$$

This equation is nonstandard due to the constraints on the structure of the matrix P and there is no known solution method. However, a parametrization approach to the synthesis of stabilizing state feedback controllers is possible.

Consider the matrix inequality

$$A^T P + PA - PBR^{-1}B^T P + Q \leq 0, \tag{23}$$

and pre- and post-multiply both sides by $X = P^{-1}$. Applying the Schur's complement formula to the result of this last step gives

$$\begin{bmatrix} AX + XA^T - BR^{-1}B^T & XQ^{1/2} \\ Q^{1/2}X & -I \end{bmatrix} \leq 0,$$
$$X = \text{diag}[X_1 \ X_2], \ X_1 > 0, \ X_2 > 0. \tag{24}$$

If the LMI (24) is feasible, then we can calculate

$$K = R^{-1}B^T P.$$

Note that (23) can be now rewritten in the form

$$(A - BK)^T P + P(A - BK) \leq -Q - K^T RK < 0$$

and hence [9] $u = -Kx$ is a state feedback control guaranteeing asymptotic stability of (10). In this case the matrices P and R play the role of parameter matrices and by varying them we obtain a set of state feedback stabilizing controllers with various properties.

IV. LQ PARAMETRIZATION OF ROBUST STABILIZING CONTROLLERS

The following results give sufficient conditions for stabilization of the uncertain continuous Roesser model considered in this paper. We begin with the following obvious result.

Lemma 3: Suppose that there exist matrices $H = \text{diag}[H_1 \ \ H_2], \ H_1 > 0, \ H_2 > 0$, and F, satisfying the following system of bilinear Lyapunov inequalities

$$(A(\delta) - B(\delta)FC)^T H + H(A(\delta) - B(\delta)FC) < 0, \ \delta \in \mathbf{\Delta}_v. \tag{25}$$

Then (5) with the control law (9) applied is asymptotically stable for all $\delta \in \mathbf{\Delta}$.

Theorem 2: There exists a gain matrix F satisfying (25) if and only if there exist matrices $H = \text{diag}[H_1 \ \ H_2] > 0$, $Q > 0$, $R > 0$, and parameter-dependent $L(\delta)$ such that the following conditions hold for all vertices $\delta \in \mathbf{\Delta}_v$

$$FC = R^{-1}(B^T(\delta)P + L(\delta)) \ , \tag{26}$$

$$A^T(\delta)H + HA(\delta) - HB(\delta)R^{-1}B^T(\delta)H + Q + L(\delta)^T R^{-1} L(\delta) \leq 0. \tag{27}$$

Proof: Necessity. Suppose that there exist matrices F, Y satisfying (25) and hence, since the inequality (25) defines a negative definite constraint on a finite set of values, this is equivalent to the existence of matrices $Q > 0$ and $R > 0$ such that

$$\begin{aligned} A_c^T(\delta)H &+ HA_c(\delta) \\ &+ Q + (FC)^T RFC < 0, \end{aligned} \tag{28}$$

where $A_c(\delta) = (A(\delta) - B(\delta)FC)$. Rearranging (28) yields

$$\begin{gathered} A^T(\delta)H + HA(\delta) + (FC)^T RFC \\ -(FC)^T B^T(\delta)H - HB(\delta)FC + Q < 0. \end{gathered} \tag{29}$$

or, on setting,

$$K(\delta) = FC - R^{-1}B^T(\delta)H \tag{30}$$

(29) can be written in the form

$$\begin{aligned} A^T(\delta)H &+ HA(\delta) - HB(\delta)R^{-1}B^T(\delta)H \\ &+ K^T(\delta)RK(\delta) + Q < 0. \end{aligned} \tag{31}$$

Define

$$L(\delta) = RK(\delta). \tag{32}$$

and by substitution in (31) we obtain (26) and (27).

Sufficiency. Suppose there exist matrices $H > 0$ and F satisfying (26) and (27). Then it follows from (26) that $L(\delta)$ and $K(\delta)$ are defined by (32) and (30) and hence using (27) we obtain

$$\begin{aligned} A^T(\delta)H &+ HA(\delta) - HB(\delta)R^{-1}B^T(\delta)H + Q \\ &+ L^T(\delta)R^{-1}L(\delta) = (A(\delta) - B(\delta)FC)^T H \\ &+ H(A(\delta) \\ &- B(\delta)FC) + (FC)^T RFC + Q < 0. \end{aligned} \tag{33}$$

Finally, it follows from this last inequality that (25) holds and therefore by Lemma 3 F is a stabilizing control law matrix. ■

This last result can be viewed as the extension of the results of [13] to $2D$ linear systems considered in this paper. There is no known method for solving the nonstandard system of coupled matrix equations and inequalities (26) and (27). Next we give sufficient conditions that are convex and lead to LMI based computation of the stabilizing control law matrix.

Set $X = H^{-1}$, $Y(\delta) = L(\delta)X$ and apply the Schur's complement formula to rewrite (26) and (27) in the form

$$\begin{bmatrix} G(\delta) & XQ^{\frac{1}{2}} & Y(\delta)^T \\ Q^{\frac{1}{2}}X & -I & 0 \\ Y(\delta) & 0 & -R \end{bmatrix} < 0, \ \delta \in \mathbf{\Delta}_v, \tag{34}$$

$$FCX = R^{-1}(B(\delta)^T + Y(\delta)), \ \delta \in \mathbf{\Delta}_v, \tag{35}$$

where $G(\delta) = XA(\delta)^T + A(\delta)X - B(\delta)R^{-1}B(\delta)$. These conditions are not convex due to the last nonlinear equality constraint, but this can be relaxed using the conservative linearizing technique of [14] which is based on the assumption that there exists a nonsingular decision variable M such that

$$CX = MC. \tag{36}$$

Hence (35) can be rewritten as

$$ZC = R^{-1}(B(\delta)^T + Y(\delta)), \ \delta \in \mathbf{\Delta}_v, \tag{37}$$

and using results in [15], (37) has a solution for $Z = FM$ if and only if

$$(B(\delta)^T + Y(\delta))(I - C^+C) = 0, \ \delta \in \mathbf{\Delta}_v, \tag{38}$$

where the superscript $+$ denotes the Moore-Penrose inverse.

If (38) holds, the gain matrix is given by

$$F = R^{-1}(B(\delta)^T + Y(\delta))C^+M^{-1} \tag{39}$$

subject to

$$B(\delta)^T + Y(\delta) = B(\gamma)^T + Y(\gamma), \ \delta, \ \gamma \in \mathbf{\Delta}_v. \tag{40}$$

Now we have the following corollary of Theorem 2.

Corollary 1: Suppose that for some matrices $Q \geq 0$, $R > 0$ the system of coupled linear equations and inequalities (34), (36), (38), (40) with respect to variables $X = \mathrm{diag}[X_1 \ \ X_2] > 0, Y(\delta)$ and M, $\delta \in \mathbf{\Delta}_v$ is feasible. Then the control law (9) with gain matrix F given by (39) ensures asymptotic stability of (5) for all $\delta \in \mathbf{\Delta}$.

The following is an algorithm for computing the stabilizing gain matrix$F$, which can be applied to a numerical example using available software such as SeDuMi [17].

Algorithm 1:

1. Assign matrices Q and r based on LQR and solve the LMI/LME problem (34), (36), (38), (40) with respect to the variables $X = \mathrm{diag}[X_1 \ \ X_2] > 0, Y(\delta)$ and M, $\delta \in \mathbf{\Delta}_v$.

2. If the LMI/LME problem of the previous step is feasible, compute the static output feedback stabilizing gain matrix F using (39).

V. NUMERICAL EXAMPLE

Consider the case of (5) when

$$A_{11} = \begin{bmatrix} 0 & 1 \\ a_1^0 & 0 \end{bmatrix}, \ A_{12} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ A_{22} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_2^0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a_3^0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & a_4^0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.92 & -1.4 & 0.92 & -1.4 \\ 0 & 0 & 0 & 0 \\ 0.65 & 1.6 & 0.65 & -1.6 \\ 0 & 0 & 0 & 0 \\ 1.4 & -1 & 1.4 & 1 \\ 0 & 0 & 0 & 0 \\ 2 & -0.8 & -2 & -0.8 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & -1.8 & 0 & 1.3 & 0 & 2.9 & 0 & 4.1 \\ 0 & -2.7 & 0 & 3.2 & 0 & -2.1 & 0 & -1.6 \\ 0 & 1.8 & 0 & 1.3 & 0 & 2.9 & 0 & -4.1 \\ 0 & -2.7 & 0 & -3.2 & 0 & 2.1 & 0 & -1.6 \end{bmatrix},$$

where $a_1^0 = -0.42,\ a_2^0 = -0.1849,\ a_3^0 = -4.41,\ a_4^0 = -4.84$. The eigenvalues of the matrices A_{12} and A_{22} are $\pm 0.6481i$ and $\pm 0.43i,\ \pm 2.1i,\ \pm 2.2i$ respectively and the nominal model is not asymptotically stable. Consider the parameter uncertainties are described by

$$a_i^0(1-\delta_i) \leq a_i^0 \leq a_i^0(1+\delta_i),\ \ i=1,\dots,4, \\ \delta_1 = 0.3,\ \delta_2=\delta_3=\delta_4=0.5, \qquad (41)$$

i.e. 30% to 50% variation is present. The resulting polytopic system for the A and B matrices in the system state-space model has $2^4 = 16$ vertices, and the problem now is to stabilize this system by applying constant static output feedback control law. To solve this problem we apply the algorithm above, we set $R = I$ and include the weighting matrix Q in the set of LMI variables. The result is

$$F = \begin{bmatrix} 0.9934 & 0.1742 & 0.0881 & 0.1148 \\ 0.1839 & 1.1862 & 0.0513 & 0.0835 \\ 0.0991 & -0.0249 & 1.1034 & -0.1981 \\ 0.0176 & 0.0234 & -0.1483 & 1.2241 \end{bmatrix},$$

with Lyapunov matrix $P = \mathrm{diag}[P_1\ P_2]$ where

$$P_1 = \begin{bmatrix} 0.0079 & 0.0009 \\ 0.0009 & 0.0146 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 3.4858 & 0.2339 & -0.4716 & 0.0018 & -0.1514 & 0.0005 \\ 0.2339 & 0.0354 & -0.0261 & 0.0107 & -0.0119 & 0.0077 \\ -0.4716 & -0.0261 & 0.6313 & 0.0403 & 0.0424 & 0.0019 \\ 0.0018 & 0.0107 & 0.0403 & 0.0276 & -0.0011 & 0.0089 \\ -0.1514 & -0.0119 & 0.0424 & -0.0011 & 0.5375 & 0.0279 \\ 0.0005 & 0.0077 & 0.0019 & 0.0089 & 0.0279 & 0.0205 \end{bmatrix}.$$

The eigenvalues of P are positive and those eigenvalues for $A_{ci}^T P + P A_{ci}$, where $A_{ci} = A_i - BCF$, are negative for all vertices. Hence by Lemma 2 the controlled system is asymptotically stable for all uncertainties given by (41). The computations required were undertaken in MATLAB using YALMIP parser [16] and the SeDuMi solver [17].

VI. CONCLUSIONS AND FURTHER WORK

The problem of robust stabilization for uncertain 2D continuous linear systems described by the Roesser state-space model has been considered with an affine parallelotopic type model of the uncertainty. Sufficient optimality conditions for an LQ state feedback controller have been developed in the absence of uncertainty and then used to describe a set of stabilizing static output feedback controllers in the presence of uncertainties of the type considered. Also an LQ parametrization of sufficient conditions for the existence of static output feedback controllers, which ensure asymptotic stability of the resulting controlled system has been obtained. These conditions are expressed in terms of LMI/LMEs and if they are feasible an explicit expression for the static output feedback controller gain matrix is available. Finally, an illustrative example has been given to illustrate the application of the results obtained. The results obtained in this paper are sufficient, and not necessary and sufficient, and hence there is conservativeness associated with them. Further research should aim to reduce the conservativeness and also extend all results to other classes of 2D linear systems.

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