Network Optimization and Synthesis using a Combined Mechanical and Electrical System: Application to Vehicle Suspension Control

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***Abstract*— This paper introduces a mechatronic network and applies it to vehicle suspensions for performance optimization. The mechatronic network consists of a ball-screw and permanent magnet electric machinery (PMEM), such that the system impedance is a combination of mechanical and electrical impedances. We then apply the network to vehicle suspensions, and demonstrate the performance benefits and their sensitivities to parameter variations. The optimal electrical impedances are constructed and experimentally verified. Based on the results, the mechatronic network is deemed effective.**

I. INTRODUCTION

From the imperfect analogy between electrical and mechanical systems, the inerter was proposed to substitute for the mass element [1], such that the reacting force of an inerter is proportional to the relative acceleration across its two terminals. With the invention of inerters, all passive mechanical networks can be realized by springs, dampers and inerters, and can be used to improve system performance in passive manners. Inerters have been applied to car suspensions [2], motorcycle steering [3], train suspensions [4], [5] and building vibration control [6]. The performance of these systems was improved by inerters, and this performance improvement can be extended by allowing higher-order impedances [4], [7]. Nevertheless, the synthesis of a high-order mechanical system is difficult due to the limitation on system volumes and weights. And system performance can also be degraded by mechanical nonlinearities [8] of complex realizations. In the present study, we proposed a novel mechatronic network that consists of a ball-screw type inerter and permanent magnet electric machinery (PMEM) [9], that realizes system impedance through a combination of mechanical elements and electrical circuits. Consequently, a high-order network can be realized by a basic mechanical structure with high-order electrical circuits, which are easier to construct than pure mechanical networks.

To demonstrate the performance benefits, we applied the mechatronic network to vehicle suspensions. First, the mechatronic network achieves better performance compared to traditional suspensions and mechanical inerter layouts. We also propose a switching control to adjust electrical circuits according to performance requirements. Second, we realize the optimal impedance using two new synthesis methods [10], [11], and show that system performance is insensitive to parameter variations. Lastly, we construct the optimal electrical circuits for experimental verification.

The paper is arranged as follows: in Section II, the mechatronic network is introduced and represented as a block diagram. Section III applies the network to vehicle suspensions to demonstrate the performance benefits. In Section IV, the optimal impedances are synthesized as electrical circuits. Section V verifies the circuit responses by experiments. Finally, we draw conclusions in Section VI.

II. MECHATRONIC NETWORK SYSTEM

The mechatronic network consists of a ball-screw and a coaxial PMEM, as shown in Fig. 1, with two terminals connected to the nut of the ball-screw set and the mount of the PMEM, respectively [9]. The relative linear motion between the nut and mount causes the rotational motion of the screw and drives the PMEM to generate a conductive voltage. Connecting suitable electrical circuits to the PMEM, we can adjust the network impedance to improve system performance.

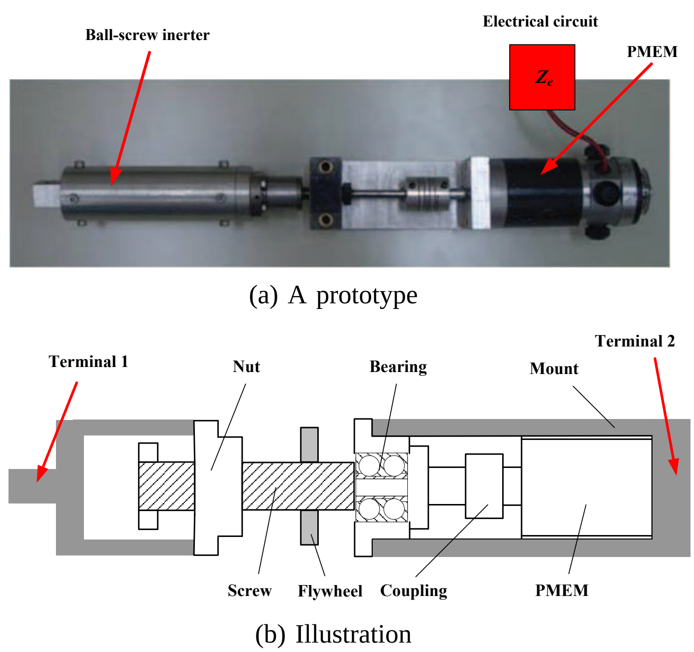

Fig. 1. The mechatronic network.[9]

As derived in [9], the impedance can be represented as a block diagram, as shown in Fig. 2, where P is pitch of the ball-screw, J is inertia of the ball-screw inerter, the armature of the PMEM is regarded as a resistor R_a in series with an

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inductor L_a, J_m and B_m represent inertia and the damping coefficient of the PMEM, respectively. k_e is the inductive voltage constant, and k_t is the inductive torque constant. Note that k_e and k_t are identical.

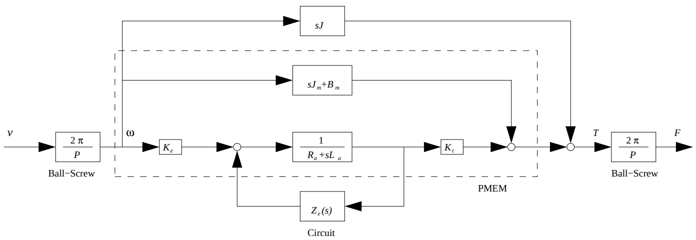


Fig. 2. The corresponding block diagram of the mechatronic network.

We can further define the network inertance $b_m = (2\pi/P)^2(J_m + J)$, damping rate $c_m = (2\pi/P)^2 B_m$, and admittance gain $K_m = (2\pi/P)^2 k_t k_e$, and represent the admittance of the mechatronic network as:

$$\frac{\hat{F}(s)}{\hat{v}(s)} = b_m s + c_m + \frac{K_m}{R_a + sL_a + Z_e(s)} = Y_{ms}. \qquad (1)$$

Note that Y_{ms} can be divided into two parts: the first part, $b_m s + c_m$, is the mechanical inerter and damper, and the second part, $K_m R_a + sL_a + Z_e(s)$, is the electrical admittance. Using the mechanical/electrical analogy, we can represent (1) as an equivalent mechanical network of Fig. 3.

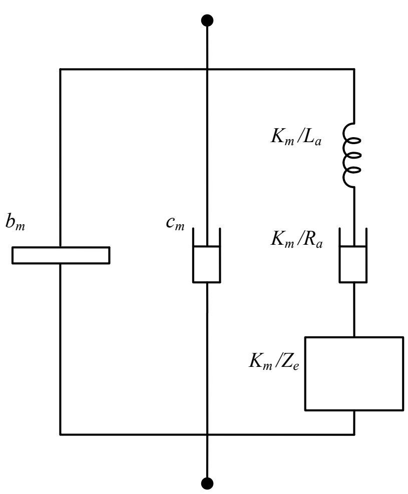


Fig. 3. The ideal mechatronic network.

III. VEHICLE SUSPENSION DESIGN

To illustrate the performance benefits, we apply the mechatronic network to a quarter-model, as shown in Fig. 4, in which the suspension force is $\hat{u}(s) = Q(s) \cdot s(\hat{z}_s - \hat{z}_u)$, where $Q(s)$ is the admittance of the suspension strut.

We consider six suspension layouts, as shown in Fig. 5, with corresponding admittances. Note that S1 is the traditional suspension, while S2 and S3 are basic parallel and serial mechanical inerter layouts. LMIS1 is a parallel

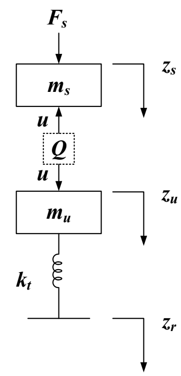
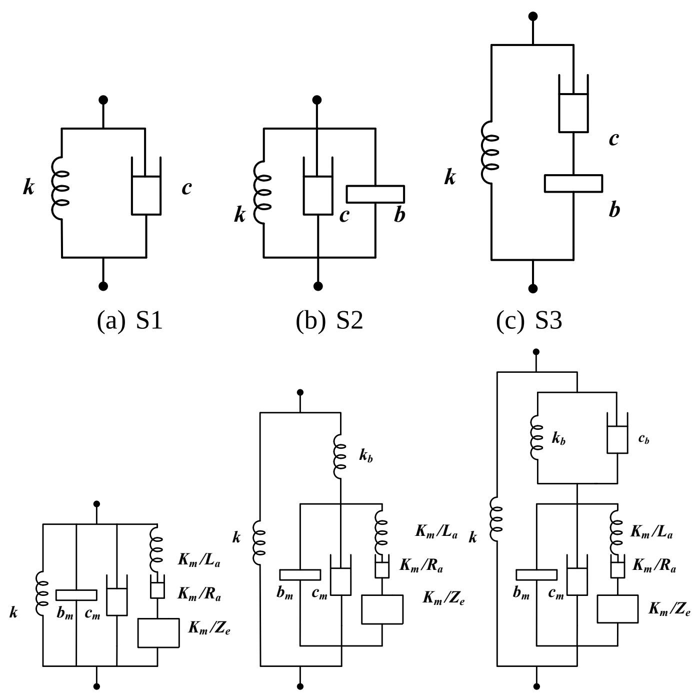


Fig. 4. The quarter-car model.

mechatronic network with a spring, while LMIS2 and LMIS3 connects the mechatronic network in series with a stiff spring k_b and a parallel spring/damper (k_b/c_b) set, respectively. To compare the performance benefits, we consider the following performance indexes [2]:

1. J_1 (ride comfort):
$$J_1 = 2\pi\sqrt{V\kappa}\|sT_{z_r\to z_s}\|_2 \qquad (2)$$
2. J_3 (dynamic tyre loads):
$$J_3 = 2\pi\sqrt{V\kappa}\|\frac{1}{s}T_{z_r\to z_s}\|_2 \qquad (3)$$

where V represents the driving velocity, κ is the road roughness parameter, $T_{z_r\to z_s}$ is the transfer function from z_r to z_s, and $\|T\|_2$ is the H_2 norm of T.



(a) S1 (b) S2 (c) S3

(d) LMIS1 (e) LMIS2 (f) LMIS3

Fig. 5. The suspension layouts.

A. Performance Optimization

Using the parameters of Table I, we optimize the performance measures J_1 and J_3 using the suspension layouts at each specified stiffness k. For S1, S2 and S3, the performance measures are convex [2] such that the optimization is global. However, the optimization of LMIS1, LMIS2 and LMIS3 is not convex because of the large number of tuning parameters. Therefore, we apply numerical optimization for these mechatronic layouts. The optimizations results are shown in Fig. 6, where LMIS3 is the best for the considered range of static stiffness. Previous studies [2], [4], [5], [8] demonstrated that mechanical inerters are particularly useful for stiffness systems, but fail to benefit soft systems. In contrast, the mechatronic network LMIS3 is shown to improve performance of both stiff and soft systems.

TABLE I
SYSTEM PARAMETERS.

Symbol	Name	Value	Unit
m_s	sprung mass	150	kg
m_u	unsprung mass	35	kg
k	static stiffness	10~120	N/mm
k_t	tire stiffness	150	N/mm
V	forward velocity	25	m/s
κ	Road roughness	5×10^{-7}	m^3 cycle^{-1}
R_a	armature resistance	2.3	Ω
L_a	armature inductance	0.7	mH
K_m	admittance gain	7056	HN/m

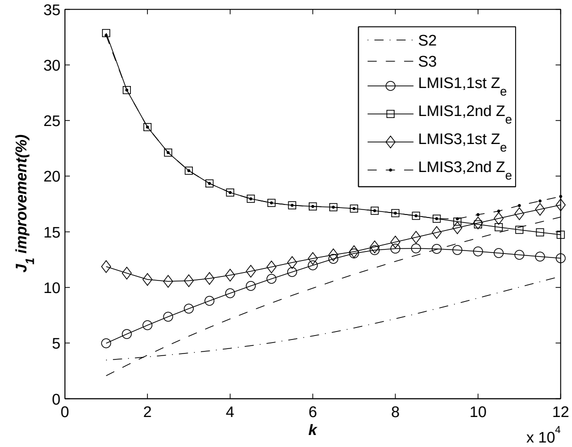
We can further define the following mixed performance index:

$$J = \beta \frac{J_1}{J_{1,0}} + (1-\beta)\frac{J_3}{J_{3,0}}, \tag{4}$$

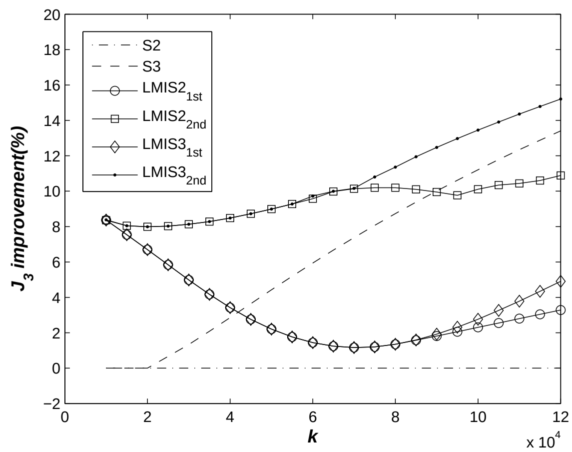
where β is a weighting, while $J_{1,0}$ and $J_{3,0}$ are optimal indexes using layout S1. Applying LMIS3 with $k = 60$ N/mm , we optimize the mixed performance for different weighting β, and illustrate the results in Table II. Note the optimization is Pareto, as shown in Fig. 7.

TABLE II
OPTIMIZATION OF MIXED PERFORMANCE.

weights β	J_1	J_3 $\times 10^2$	J	% improve of J_1	% improve of J_3
0		4.593	0.904		9.578
0.1	1.706	4.603	0.906	3.050	9.381
0.2	1.668	4.624	0.910	5.248	8.969
0.3	1.586	4.720	0.929	9.908	7.083
0.4	1.548	4.766	0.938	12.050	6.179
0.5	1.523	4.829	0.951	13.461	4.938
0.6	1.505	4.899	0.964	14.494	3.560
0.7	1.490	4.987	0.982	15.348	1.821
0.8	1.477	5.112	1.006	16.088	-0.632
0.9	1.465	5.329	1.049	16.752	-4.912
1.0	1.456		1.189	17.281	



(a) Percentage improvement of J_1



(b) Percentage improvement of J_3

Fig. 6. Percentage improvement of J_1 and J_3.

B. Circuit Switching Control

Because it is much easier to switch electrical circuits than mechanical networks, we can switch the circuits according to performance requirements. For example, when $\beta = 0.5$, the optimal mechanical components are found as: $b_m = 15.58$ kg, $c_m = 0$, $c_b = 2.832 \times 10^6$ Ns/m, $k_b = 5.493 \times 10^8$ N/mm. Using these mechanical components, we optimize the performance measures J_1 and J_3 with the following two circuits:

$$Z_e^{J_1} = \frac{3.934s^2 + 33.53s + 1676}{s^2 + 54.32s + 0.01381}, \tag{5}$$

$$Z_e^{J_3} = \frac{2568s^2 + 5415s + 2.403 \times 10^6}{s^2 + 1.059 \times 10^5 s + 0.01361}, \tag{6}$$

That is, we can switch the circuits for specified performance requirements. From Table II and Fig. 7, the switched control can improve J_1 from 1.523 to 1.508 with $Z_e^{J_1}$, and improve J_3 from 4.829$\times 10^2$ to 4.753$\times 10^2$ with $Z_e^{J_3}$.

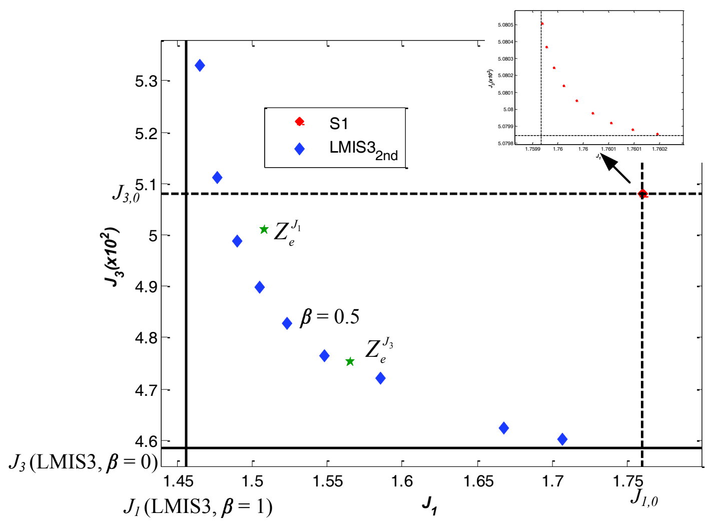


Fig. 7. Multi-objective optimization.

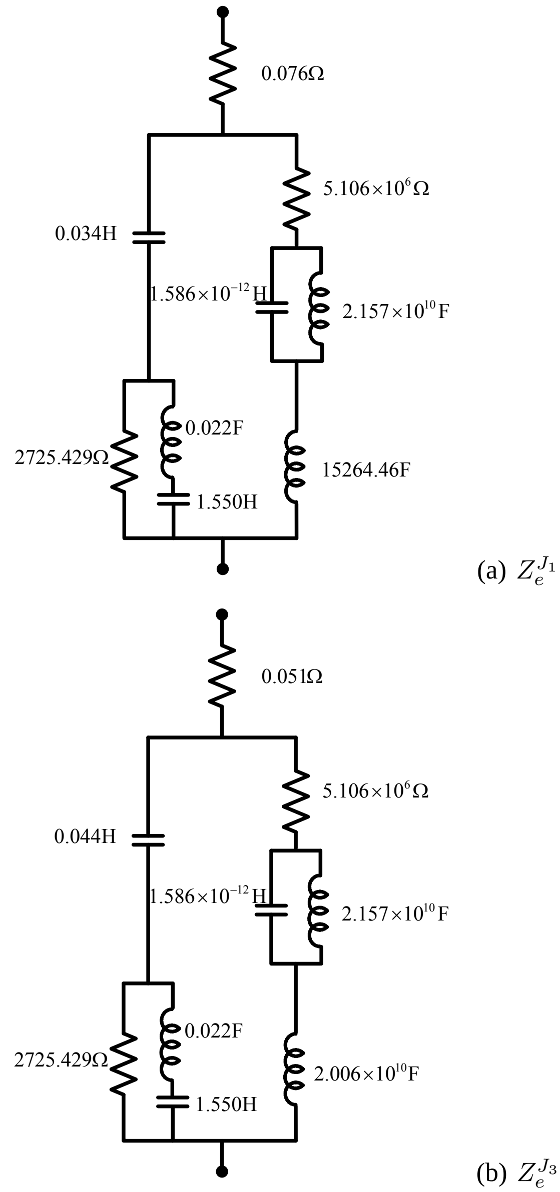


Fig. 8. Butt-Duffin realization of $Z_e^{J_1}$ and $Z_e^{J_3}$.

IV. NETWORK SYNTHESIS AND PERFORMANCE SENSITIVITY

To synthesize the electrical impedance $Z_e^{J_1}$ and $Z_e^{J_3}$, we can use Brune or Butt-Duffin methods [12]. However, Brune realization requires perfect transformers, and Butt-Duffin realization uses a large amount of elements. For example, $Z_e^{J_1}$ and $Z_e^{J_3}$ can be synthesized as Fig. 8, where nine elements are used for a second-order impedance (with five parameters) and the component values are unreasonable.

Two new network syntheses were proposed recently. Chen and Smith [10] use a *paramount* matrix to develop a sub-class of second-order network that contains four resistors, one capacitor and one inductor. The synthesis was further simplified in [11], where a *regular* second-order positive-real function can have a network synthesis with three resistors, one capacitor and one inductor. For example, $Z_e^{J_1}$ and $Z_e^{J_3}$ can be synthesized as in Fig. 9, with the following values:

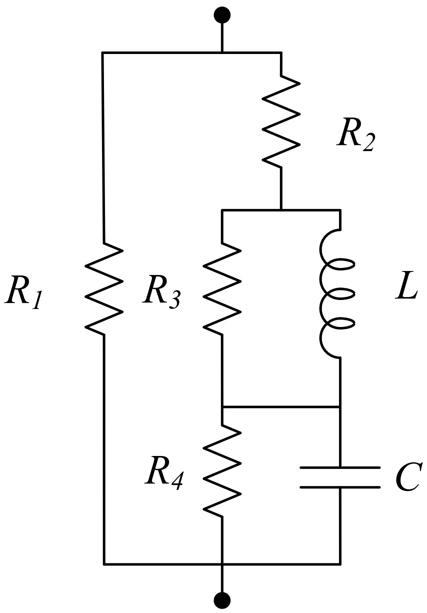
1. Chen-Smith synthesis:

$$\begin{aligned} Z_e^{J_1} \quad : \quad & R_1 = 3.963 \times 10^6\Omega, R_2 = 0.0492\Omega, \\ & R_3 = 3.885\Omega, R_4 = 1.252 \times 10^5\Omega, \\ & C = 0.0324\text{F}, L = 0.0715\text{H}. \\ Z_e^{J_3} \quad : \quad & R_1 = 3.887 \times 10^9\Omega, R_2 = 0.0509\Omega, \\ & R_3 = 2567.951\Omega, R_4 = 1.850 \times 10^8\Omega, \\ & C = 0.0441\text{F}, L = 0.0242\text{H}. \end{aligned}$$

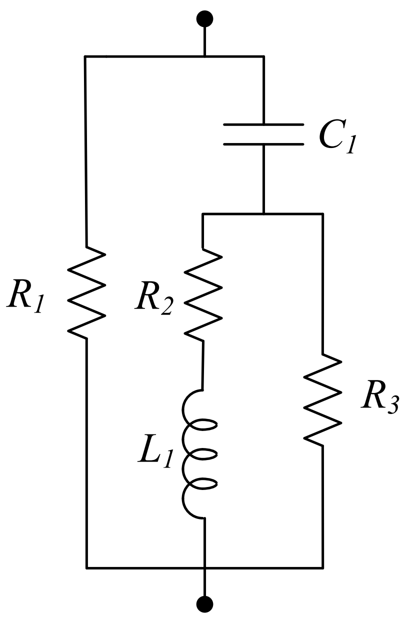
2. Jiang-Smith synthesis:

$$\begin{aligned} Z_e^{J_1} \quad : \quad & R_1 = 1.214 \times 10^5\Omega, R_2 = 0.0499\Omega, \\ & R_3 = 3.934\Omega, \\ & C = 0.0324\text{F}, L = 0.0733\text{H}. \\ Z_e^{J_3} \quad : \quad & R_1 = 1.766 \times 10^8\Omega, R_2 = 0.051\Omega, \\ & R_3 = 2568.037\Omega, \\ & C = 0.0441\text{F}, L = 0.0242\text{H}. \end{aligned}$$

Considering the realization of electrical components (especially capacitors and inductors), we further discuss performance sensitivity to parameter variations. The results are illustrated in Fig. 10, where the capacitors and inductors are set as ± 20% of the optimal values. From Fig. 10, the performance is only slightly decreased by these component variations. For example, J_1 is degraded from 1.508 to 1.531 (or 1.518) when C_1 is changed from 0.0324H to 0.026H (or 0.039H). And J_3 is degraded from 4.753 $\times 10^2$ to 4.782 $\times 10^2$ (or 4.766 $\times 10^2$) when C_1 is varied from 0.0441H to 0.035H (or 0.053H). That is, compared to the optimal J_1 and J_3, they are only degraded by less than 2% even when the parameters are varied by 20%.

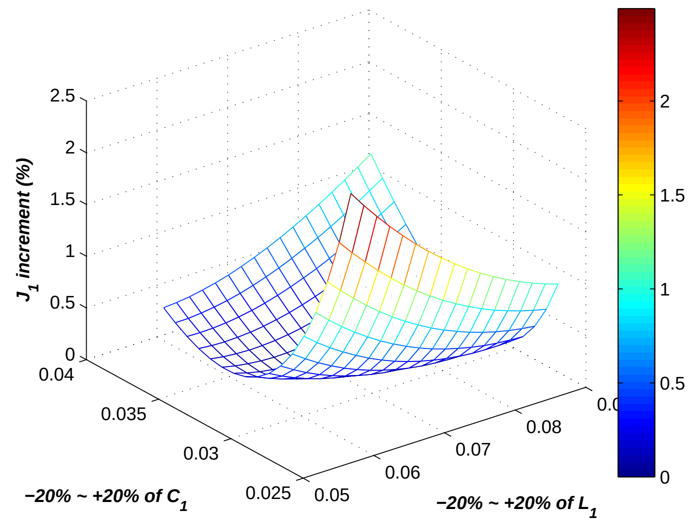


(a) Chen-Smith synthesis

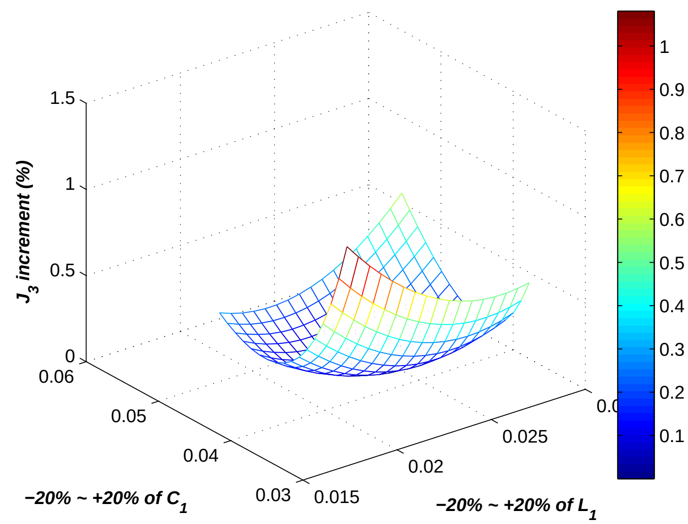


(b) Jiang-Smith synthesis

Fig. 9. Network synthesis for a second-order impedance.



(a)J_1 with 20% L_1 and C_1 variations.



(b)J_3 with 20% L_1 and C_1 variations.

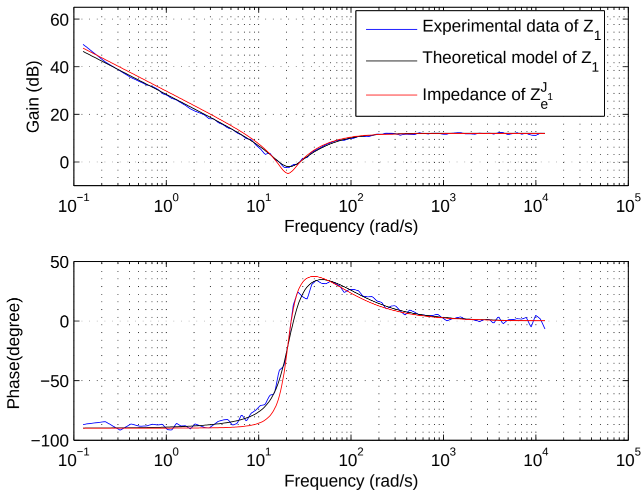
Fig. 10. Performance sensitivities.

V. EXPERIMENTAL RESULTS

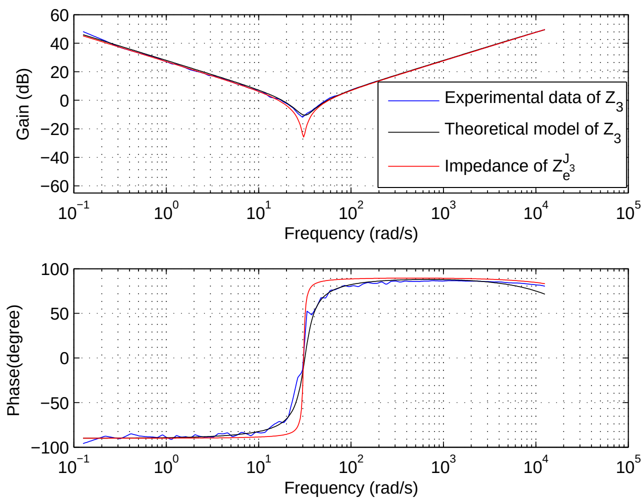
To verify the system responses, we construct two electrical impedances from Fig. 9(b) synthesis with the the following components:

1. Z_1: $R_1 = 8.2258 \times 10^6, R_2 = 0.5\Omega, R_3 = 4\Omega, C = 0.038\text{F}, L = 0.066\text{H}$, for
$$Z_1 = \frac{4s^2 + 56.62s + 1794}{s^2 + 68.18s + 2.181 \times 10^{-4}},$$
which gives $J_1 = 1.530$ (compared to $J_1 = 1.508$ with $Z_e^{J_1}$).
2. Z_2: $R_1 = 1.056 \times 10^6, R_2 = 0.3\Omega, R_3 = 940\Omega, C = 0.04\text{F}, L = 0.025\text{H}$, for
$$Z_2 = \frac{8.431 \times 10^5 s^2 + 1.265 \times 10^7 s + 8.783 \times 10^8}{s^2 + 4.216 \times 10^7 s + 831.7},$$
which gives $J_3 = 479.3$ (compared to $J_3 = 475.3$ with $Z_e^{J_3}$).

Note that Z_1 and Z_2 are used to substitute for $Z_e^{J_1}$ and $Z_e^{J_3}$, respectively, due to the limitation on available electrical elements. Using Agilent 35670A, we generate an input signal to the circuits and measure the output responses. Applied identification techniques [13], the results are shown in Fig.11, where the experimental responses are closed to the theoretical responses. Therefore, they can be directly applied to the mechatronic network.



(a)Z_1.



(b)Z_2.

Fig. 11. Frequency responses of the electrical circuits.

VI. CONCLUSIONS

The paper has demonstrated a mechatronic network, whose impedance can be easily adjusted by electrical circuits. We applied it to vehicle suspensions, and showed that the mechatronic network can achieve significant performance benefits for both stiff and soft systems. In addition, different performance requirements can be achieved by switching circuits. We then applied two new synthesis methods to realize the optimal impedances, and illustrated that the performance indexes were insensitive to component variations. Lastly, we constructed an electrical network for the optimal impedance, and verified the system responses by experiments.

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