

Redundancies in Transformerless Network Synthesis

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Abstract—The purpose of this paper is to give a compact summary of recent results based on the concept of a regular positive-real function. We will list an efficient set of networks which is capable of realising all biquadratics which are realisable by 5-element networks or 6-element networks of series-parallel type. The structures are simpler than the full Bott-Duffin synthesis, though there are some (non-regular) positive-real biquadratics which cannot be realised by this class.

I. INTRODUCTION

It is well known that any positive-real function $Z(s)$ can be realised as the driving-point immittance of a circuit comprising resistors, capacitors and inductors only [1]. When $Z(s)$ is a biquadratic, the circuit realisation takes the form shown in Fig. 1. These realisations contain six reactive elements and three resistors. Subsequently, a simplification was found by several authors independently [2], [3], [4], [5], [6] to remove one of the reactive elements. To date it is not known if this is the “simplest possible” realisation. The “apparent redundancy” of the construction has subsequently intrigued many researchers. Following the work of [1] a number of authors attempted to classify networks of low complexity in an attempt to find the simplest and most powerful realisations [7], [8], [9], [10], [11], [12], [13], [14]. Work on this topic ceased in the early 1970s without a complete picture being obtained.

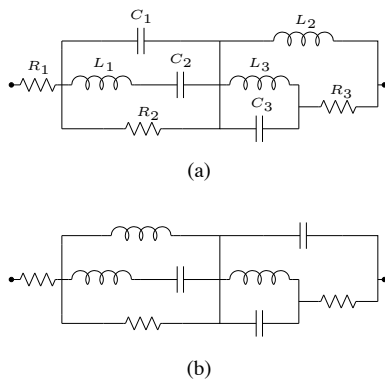


Fig. 1. The Bott-Duffin realisation for positive-real biquadratics

Recently, a new network element (the inerter) was introduced for mechanical control [15] which has revived interest in passive network realisations. The inerter is a mechanical two-terminal element with the property that the applied force at the terminals is proportional to the relative acceleration

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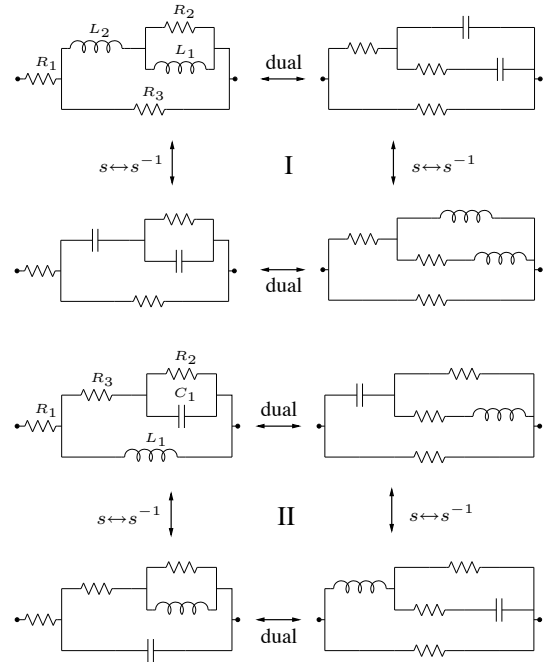


Fig. 2. The two series-parallel five-element network quartets with two reactive elements that can realise all the regular biquadratics.

between the terminals. Applications of the method to vehicle suspension [16], [17], control of motorcycle steering instabilities [18], [19] and vibration absorption [15] have been identified. The inerter has been successfully deployed in Formula One racing since 2005 [20]. Independently, the need for a renewed attack on the subject of passive network synthesis has been advocated by Kalman [21].

This paper summarises the recent work of the authors based on the concept of a regular positive-real function. In particular, we will list an efficient set of networks which is capable of realising all biquadratics which are realisable by 5-element networks or 6-element networks of series-parallel type. This set is a strict subset of all the biquadratic positive-real functions.

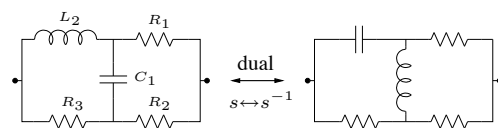


Fig. 3. The two-reactive five-element bridge network quartet that can realise some non-regular biquadratics.

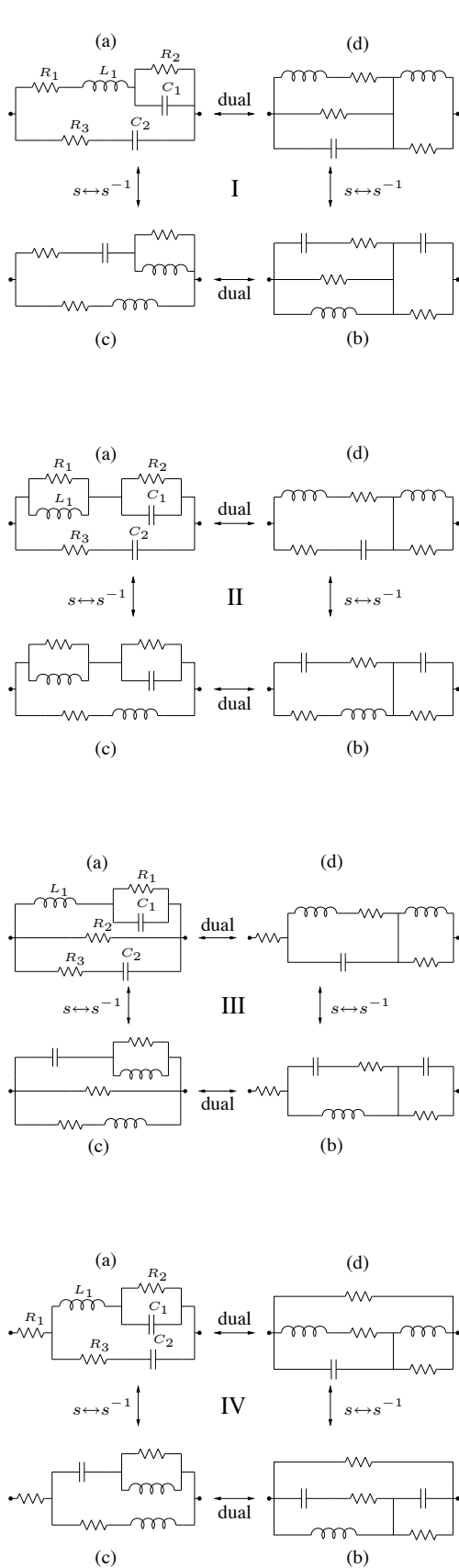


Fig. 4. The four network quartets of the series-parallel three-reactive six-element networks that can realise some non-regular biquadratics.

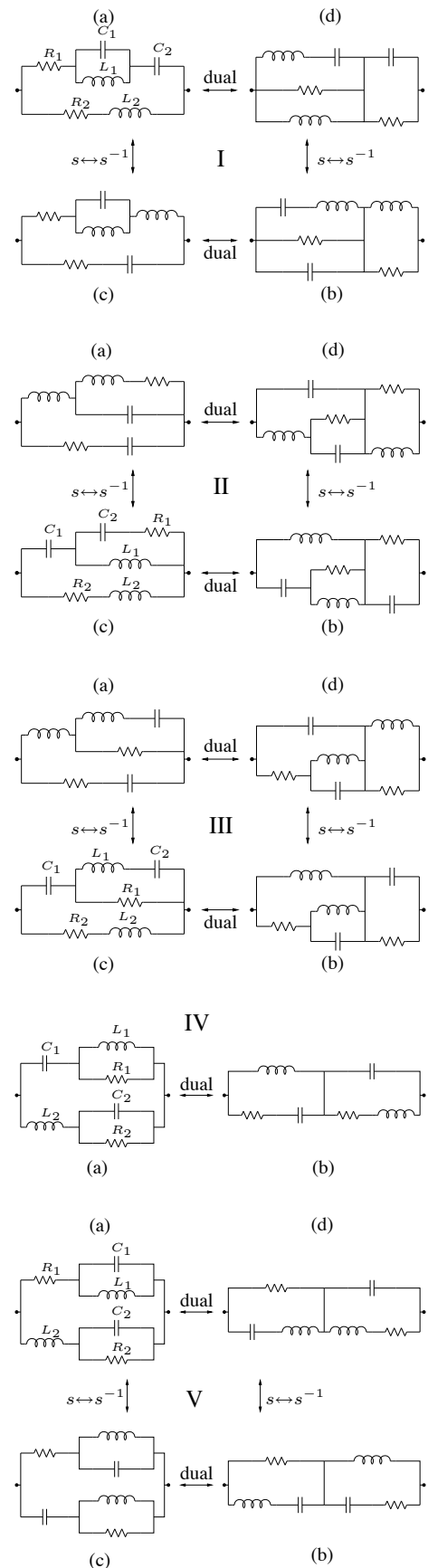


Fig. 5. The five network quartets of the series-parallel four-reactive six-element networks that can realise some non-regular biquadratics.

II. SUMMARY

The concept of a regular positive-real function was introduced in [22]: a positive-real function $Z(s)$ is defined to be *regular* if the smallest value of $\operatorname{Re}(Z(j\omega))$ or $\operatorname{Re}(Z^{-1}(j\omega))$ occurs at $\omega = 0$ or $\omega = \infty$. A series of lemmas was introduced in [22] characterising the basic properties of regularity. This concept has been shown to be useful in the classification of low-complexity two-terminal networks.

The class of the positive-real biquadratic functions which can be realised by 5-element networks was studied in [22]. In particular, it was shown that a biquadratic can be realised by a series-parallel network with two reactive elements if and only if it is regular. Moreover, there are two “network quartets” which can realise all regular biquadratics, shown in Fig. 2. Furthermore, it was shown that bridge networks with two reactive and three resistive elements can only realise regular immittances except for the network quartet of Fig. 3.

In [23] the series-parallel 6-element networks with three reactive elements were investigated. Vasiliu [13] claims that there are in total 16 series-parallel 6-element networks with three reactive elements, which can realise biquadratic immittances that otherwise would require a full Bott-Duffin synthesis. In [23] a classification procedure to find an efficient subset of such networks which may realise any non-regular biquadratic that can be synthesised by this class of networks was described. Four network quartets shown in Fig. 4 were identified which serve this purpose. The realisable conditions for each of the networks shown in Fig. 4 have also been provided in [23].

In [24] the series-parallel 6-element networks with four reactive elements were investigated. In [13] Vasiliu claims there are only six series-parallel networks with four reactive elements which can realise classes of biquadratic immittances that otherwise would require a full Bott-Duffin synthesis. Vasiliu [25] later claims that there are 12 more such networks. In [24] the regularity concept was used to find an efficient subset of such networks which may realise any non-regular biquadratic that can be synthesised by this class of networks. Five network quartets shown in Fig. 5 were identified which serve this purpose (one of which contains only two networks), independently confirming the results of Vasiliu [13], [25].

III. CONCLUSIONS

In [1] it was shown how to realise any positive-real functions using resistors, capacitors and inductors only. Fig. 1 shows the construction for a biquadratic. Figs. 2-5 give an efficient set of networks which may realise any biquadratic that can be realised by 5-element networks or 6-element series-parallel networks. Not all positive-real biquadratics fall within this class. It is currently unknown if a simpler construction than [1]-[6] is available to realise all positive-real (biquadratic) functions.

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