Output Feedback Stabilizing Control and Passification of Switching Diffusion Systems

Pavel Pakshin and Dimitri Peaucelle

***Abstract*— A parametric description of static output feedback stabililizing controllers for diffusion systems with Markovian switching is presented. This description is expressed in terms of coupled linear matrix equations and non-convex quadratic matrix inequalities which depend on parameter matrices similar to weight matrices in LQR theory. A convexifying approximation technique is proposed to obtain the LMI-based algorithms for computing of the gain matrix. These are non-iterative and used computationally efficient SDP solvers.The results are then applied to simultaneous stabilization of a set of diffusion systems, robust stabilization and stochastic passification problems. Finally, a numerical example is provided to demonstrate the applicability and effectiveness of the proposed method.**

I. INTRODUCTION

The problem of stabilization via static output feedback is a tricky problem in the modern control theory. On the one hand there exists a lot of necessary and sufficient conditions of stabilization on the other hand these conditions are hard to implement and can impose very difficult numerical problems. A survey of static output-feedback control is given in [26]. A lot of work has been pursued after publication of this survey [4], [7], [23], [29] and references therein, however several problems are still open. Moreover there exist few results concerning these problems for the class of stochastic systems [19] and references therein. Main objective of this paper is to present a LQR type parameterization of static output feedback controllers for continuous-time diffusion systems with Markovian switching. That parameterization is derived from classical LQR parameterizations of state-feedback controllers with restrictions to have an output-feedback structure [7]. Based on this parametrization a new approach to design of static output feedback stabilizing control is developed. This approach leads to algorithms for computation of stabilizing gain which may be implemented with existing semi-definite solvers such as SeDuMi [25] and easily coded in Matlab environment using YALMIP [13]. It turns out that particular cases of obtained result give effective solution for simultaneous stabilization, robust stabilization and passification via static output feedback.control.

The passification property plays important role in simple adaptive control schemes [2], [11] and it is as follows: there should exist a stabilizing static output feedback and linear combination of the output measurement vector y and the reference signal w (called passive output z) such that the closed-loop is passive with respect to the couple w, z. This property has been for long studied assuming that the passive output is the measured vector $z = y$. In such case the passification property is also called ”almost passivity” [3], i.e. passive at the expense of finding a static output-feedback. But having $z = y$ limits the method to square systems (same number of inputs and outputs). Therefore extensions have been proposed [6] to the case when $z = Gy$ for a given G. Then in [20], [21] the issue of robust passivity-based adaptive control was proved to have solutions only if $z = Gy + Dw$, that is if one allows a feed-though gain in the definition of z. That results was largely inspired by the results on ”schunting” the original system for obtaining passification property [11].

But in all these results, the matrix G is assumed to be known, obtained by some physical considerations or by other means. Finding this G matrix happens to be a highly complex problem that is as difficult as finding a stabilizing static output-feedback. We shall proceed in two steps to design the matrix G. First, recalling that strict passivity implies stability, we shall design a robustly stabilizing feedback gain F. Second, for that gain F we shall design the matrix G. This approach was developed by the authors for uncertain deterministic systems [15] for stochastic diffusion systems [16] and for systems with Markovian switching [17]. This paper extends the mentioned results to the class of diffusion systems with Markovian switching. In the past years there has been a growing interest in the study of this class of systems [10], [28].

The LMI results that we propose do not claim to be to solve all possible stabilization and passification problems. We have adopted a LMI based strategy using either the conservative linearizing relaxation of [4] or sufficient conditions [15] based on convex properties of Riccati inequality [1]. The strategy moreover needs to initialize *a priori* the R, Q matrices of some LQR problem. While this last issue has in practice some interesting properties as shown in [15], [16], the former is clearly the major limitation. In order to avoid such conservative restrictions one would need to solve the original problem with non convex optimization techniques such as those developed in HIFOO [8] or using a solver for bilinear matrix inequalities such as PenBMI [9]. Such tools would probably give improved results compared to those exposed here. Yet our aim is not a full numerical comparison but the exploration of possible new paths for the exposed

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stabilization problem.

The paper is organized as follows. In the next section we formulate the problems to be solved. In section III the main LQR parametrization theorem for diffusion systems with Markovian switching is given. Then in the next section convex sufficient conditions are obtained and LMI based algorithms for computation of stabilizing gain are proposed. In section V the results are applied to simultaneous and robust stabilization problems. The section VI is devoted to the design of passifying output. Section VII illustrates the results on a numerical example.

II. PROBLEM DESCRIPTION

A. Main notations

We use the following notations: $\mathbb{R}^n$ is the set of real n dimensional vectors as $\mathbb{R}^{m\times p}$ is the set of $m\times p$ real matrices, A^T is the transpose of the matrix, A, $\mathbf{1}$ and $\mathbf{0}$ are respectively the identity and the zero matrices of appropriate dimensions. For Hermitian matrices, $A > (\geq) B$ means that $A - B$ is positive (semi) definite. A^{-1} is the inverse of A when it exists while A^+ is the Moore-Penrose inverse.

B. Statement of the problem

Consider switching diffusion system [10] described by the following equations

$$dx(t) = [A(r(t))x(t) + B(r(t))u(t)]dt$$
$$+\sum_{l=1}^{m} \gamma_l(r(t))[A_l(r(t))x(t) + B_l(r(t))u(t)]dw_l(t), \quad (1)$$
$$y(t) = C(r(t))x(t),\ t \geq 0,$$

where $x(t) \in \mathbb{R}^{n_x}$ is the continuous component of the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input vector; $y(t) \in \mathbb{R}^{n_y}$ is the output vector; $r(t)$ $(t \geq 0)$ is the discrete component of the state, taking values in a finite set $\mathbb{N} = \{1, \ldots, N\}$; this component is modeled by homogeneous Markov chain with the mode transition probabilities

$$\mathrm{Prob}(r(t+\Delta t) = j | r(t) = i)$$
$$= \begin{cases} \pi_{ij}h + o(\Delta t), & \text{if} \quad j \neq i, \\ 1 + \pi_{ii}h + o(\Delta t), & \text{if} \quad j = i, \end{cases} \quad (2)$$

$i,\, j \in \mathbb{N}$, $\pi_{ij} > 0$ $(i \neq j)$ denotes the switching rate from mode i at time t to mode j at time $t + \Delta t$ for $\Delta t > 0$ and $\pi_{ii} = -\sum_{i\neq j}^{N} \pi_{ij}$; the Markov chain transition rates matrix is defined by $\Pi = [\pi_{ij}]_1^N$; $\gamma_l(\cdot)$ $(l = 1, \ldots, m)$ are positive scalars; $w(t) = [w_1(t) \ldots w_m(t)]^{\mathrm{T}}$ is the $\mathbb{R}^m$-valued standard Wiener process defined on the probability space $(\Omega, \mathcal{F}, \mathrm{P})$ with natural filtration $\{\mathcal{F}_t\}$; for $r(t) \in \mathcal{N}$ the system matrices and scalar parameters of the i-th mode are denoted by $A_i, B_i, A_{li}, B_{li}, C_i$ and γ_{li} which are real known with appropriate dimensions; the initial conditions $x(0) = x_0, r(0) = r_0$ are deterministic.

The two-component process $[x(\cdot)\ r(\cdot)]^{\mathrm{T}}$ in the hybrid space $\mathbb{R}^{n_x} \times \mathbb{N}$ satisfying (1), (2) is termed a switching diffusion or a mode(regime)-switching diffusion. The components $x(t)$ and $r(t)$ are called continuous and discrete ones corresponding to their sample path properties. It is assumed that at the moment $\tau > 0$ of the mode change the continuous component can be changed by jump

$$x(\tau) = \Phi_{ij}x(\tau - 0),\ i, j \in \mathbb{N}, \quad (3)$$

where Φ_{ij} $(i, j \in \mathbb{N})$ are constant matrices.

Let the switching static output feedback control has the form

$$u(t) = -F_i y(t),\ \text{if}\ r(t) = i,\ i \in \mathbb{N}. \quad (4)$$

Definition 1: [10] The system (1),(4) is said to be exponentially stable in the mean square if for all pairs $x_0 \in \mathbb{R}^n,\ r_0 \in \mathbb{N}$ there exist constant scalars $\beta > 0,\ \alpha > 0$ such that

$$\mathrm{E}[\| x(t) \|^2 | \, x(0) = x_0,\ r(0) = r_0] \leq \beta \parallel x_0 \parallel^2 \exp(-\alpha t),$$

where $E(\cdot)$ is the expectation operator and $\| \cdot \|$ denotes the Euclidian norm.

The purpose of the paper is to describe in in terms of the LQR-type parameters all the gain matrices in (4), such that the system (1), is exponentially stable in the mean square (ESMS) and to derive LMI based algorithm for computing these gain matrices. Applications of the obtained results to simultaneous stabilization, robust stabilization and passification problems will be also considered. The paper extends to the class of switching diffusion systems the results obtained by the authors in [15], [16], [17].

III. PARAMETRIZATION OF ALL STABILIZING GAINS

The following theorem gives parametric description (parametrization) of all stabilizing static output feedback gains. This theorem is based on LQR concept and extends the results of [7], [15], [16], [17] to the considered class of switching diffusion systems.

Theorem 1: There exists a gain matrix F_i such that the system (1), (4) is exponentially stable in the mean square if and only if there exist parameter matrices $Q_i = Q_i^T > \mathbf{0},\ R_i = R_i^T > \mathbf{0}$ such that

$$F_i C_i = [R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i],\ i \in \mathbb{N}, \quad (5)$$

where $P_i = P_i^T > \mathbf{0}$ and L_i $(i \in \mathbb{N})$ is a solution to the system of coupled matrix inequalities

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}} P_i$$
$$+\Theta_i(P_i)^{\mathrm{T}}] + Q_i + \Delta_i(P_i) + \Psi_i(P_1, \ldots, P_N)$$
$$+L_i^{\mathrm{T}}[R_i + \Gamma_i(P_i)]^{-1} L_i \leq \mathbf{0},\ i \in \mathbb{N}, \quad (6)$$

$$\Gamma_i(P_i) = \sum_{l=1}^{m} \gamma_{li}^2 B_{li}^{\mathrm{T}} P_i B_{li},\ \Delta_i(P_i) = \sum_{l=1}^{m} \gamma_{li}^2 A_{li}^{\mathrm{T}} P_i A_{li},$$
$$\Theta_i(P_i) = \sum_{l=1}^{m} \gamma_{li}^2 A_{li}^{\mathrm{T}} P_i B_{li},$$
$$\Psi_i(P_1, \ldots, P_N) = \sum_{j\neq i}^{N} \pi_{ij}[\Phi_{ij}^{\mathrm{T}} P_j \Phi_{ij} - P_i],\ i \in \mathbb{N}.$$

Proof: *Necessity.* Let matrix F_i be a stabilizing gain. Then according to [10] there exists a positive definite solution $P_i = P_i^T$ to the system of inequalities

$$A_{ci}^{\mathrm{T}} P_i + P_i A_{ci} + \Psi_i(P_1, \ldots, P_N) + \sum_{l=1}^{m} \gamma_{li}^2 (A_{li} - B_{li} F C)^{\mathrm{T}} P_i (A_{li} - B_{li} F C) + Q_i + (F_i C_i)^T R_i F_i C_i \leq \mathbf{0},\ i \in \mathbb{N} \quad (7)$$

where $A_{ci} = (A_i - B_i F_i C_i)$, $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$ and $R_i = R_i^{\mathrm{T}} > \mathbf{0}$.

Rearranging (7) yields

$$A_i^{\mathrm{T}} P_i + P_i A_i + (F_i C_i)^{\mathrm{T}} (R_i + \Gamma_i(P_i) F_i C_i - (F_i C_i)^{\mathrm{T}} (B_i^{\mathrm{T}} P_i + \Theta_i^{\mathrm{T}}(P_i)) - (P_i B_i + \Theta_i(P_i)) F_i C_i + \Delta_i(P_i) + \Psi_i(P_1, \ldots, P_N) \leq \mathbf{0},\ i \in \mathbb{N}. \quad (8)$$

Set

$$K_i = F_i C_i - (R_i + \Gamma_i(P_i))^{-1} (B_i^T P_i + \Theta_i^{\mathrm{T}}(P_i)) \quad (9)$$

and rewrite (8) in the following form

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1} [B_i^{\mathrm{T}} P_i + \Theta_i^{\mathrm{T}}(P_i)] + K_i^{\mathrm{T}} [R_i + \Gamma_i(P_i)] K_i + \Delta_i(P_i) + \Psi_i(P_1, \ldots, P_N) + Q_i \leq \mathbf{0}. \quad (10)$$

Define next

$$L_i = [R_i + \Gamma_i(P_i)] K_i. \quad (11)$$

Substituting (11) in (10) we easily obtain (5) and (6).

Sufficiency. Let there exist matrices $P_i = P_i^{\mathrm{T}}$ and F_i satisfying (5) and (6). It follows from (5) that L_i and K_i are defined by (11) and (9), then from (6) we obtain

$$\begin{aligned} \mathbf{0} \geq{}& A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1} [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + Q_i + \Delta_i(P_i) \\ &+ \Psi_i(P_1, \ldots, P_N) + L_i^{\mathrm{T}} [R_i + \Gamma_i(P_i)]^{-1} L_i \\ ={}& A_{ci}^{\mathrm{T}} P_i + P_i A_{ci} + \Psi_i(P_1, \ldots, P_N) + \sum_{l=1}^{m} \gamma_{li}^2 (A_{li} - B_{li} F C)^{\mathrm{T}} P_i (A_{li} - B_{li} F C) \\ &+ Q_i + (F_i C_i)^T R_i F_i C_i,\ i \in \mathbb{N}. \end{aligned} \quad (12)$$

Because $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ it follows from (12) that (7) holds and according to [10] the matrix F_i is a stabilizing gain. The proof is complete. ∎

IV. CONVEX SUFFICIENT CONDITIONS AND ALGORITHMS

As for the general static output feedback design, there is no known exact convex methodology for the design of the gain matrix F_i $(i \in \mathbb{N})$. Based on existing convexifying techniques, we provide now two conservative LMI based results for the problem. Each of these techniques may possibly fail even if stabilizing gains exist, yet in practice, one or the other, happens to be successful on examples.

A. Convex approximation I

Assume given matrices Q_i, R_i and $L_i\,(i \in \mathbb{N})$ and let a scalar $\mu_i (i \in \mathbb{N})$ sufficiently large for the following inequality to hold

$$\begin{bmatrix} \mu_i Q_i & L_i^{\mathrm{T}} \\ L_i & R_i + \Gamma_i(P_i) \end{bmatrix} > \mathbf{0},\ i \in \mathbb{N}. \quad (13)$$

Assume as well $P_i = P_i^{\mathrm{T}} > 0$ $(i \in \mathbb{N})$ solution to the coupled Riccati equations

$$A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1} [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + \Delta_i(P_i) + \Psi_i(P_1, \ldots, P_N) + (1 + \mu_i) Q_i = \mathbf{0}. \quad (14)$$

Taking into account (6) we easily obtain from (13):

$$\begin{aligned} & A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1} [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}}] + Q_i \\ & + \Delta_i(P_i) + \Psi_i(P_1, \ldots, P_N) + L_i^{\mathrm{T}} [R_i + \Gamma_i(P_i)]^{-1} L_i \\ & \leq A_i^{\mathrm{T}} P_i + P_i A_i - [P_i B_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1} [B_i^{\mathrm{T}} P_i \\ & + \Theta_i(P_i)^{\mathrm{T}}] + \Delta_i(P_i) + \Psi_i(P_1, \ldots, P_N) + (1 + \mu_i) Q_i = \mathbf{0}. \end{aligned}$$

The equation (5) has exact solution with respect to gain matrix only for special form of the right hand side. According to [24] this equation is solvable with respect to if and only if

$$[B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i](\mathbf{1} - C_i^{+} C_i) = \mathbf{0}, \quad (15)$$

Moreover the solution of (5) is then given by

$$F_i = [R_i + \Gamma_i(P_i)]^{-1} [B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i] C^{+}. \quad (16)$$

These conditions can be also formulated in terms of singular value decomposition of the output matrix C_i [7], [29], [19].

So we have the following result

Corollary 1: Let for some scalar $\mu_i > 0$ and parameter matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ $(i \in \mathbb{N})$ the system of coupled Riccati equations (14) has positive definite solution $P_i = P_i^{\mathrm{T}} > \mathbf{0}$ satisfying (13), (15) for some matrix L_i $(i \in \mathbb{N})$. Then the control law (4) with the gain matrix given by (16) provides ESMS of the system (1).

Based on convex sufficient conditions of Corollary 1 and LMI based method to solution of Riccati equation [1] we can formulate the following algorithm for the design of stabilizing gains F_i.

Algorithm 1:

1. Assign matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ $(i \in \mathbb{N})$, scalar $\mu_i > 0$ based on LQR reasons and solve the following LMI optimization problem with respect to variables $P_i = P_i^{\mathrm{T}} > \mathbf{0}$ and L_i:

$$\mathrm{Tr} \sum_{j=1}^{N} P_j \to \max$$

$$\begin{bmatrix} \Upsilon_i(P_1, \ldots, P_N) & P_i B_i + \Theta_i(P_i) \\ B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} & \Gamma_i(P_i) + R_i \end{bmatrix} \geq \mathbf{0},$$

$$\begin{bmatrix} \mu_i Q_i & L_i^{\mathrm{T}} \\ L_i & \Gamma_i(P_i) + R_i \end{bmatrix} > \mathbf{0},$$

$$(B_i^{\mathrm{T}} P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i)(\mathbf{1} - C_i^{+} C_i) = \mathbf{0},$$

where

$$\Upsilon_i(P_1,\dots,P_N) = A_i^{\mathrm{T}}P_i + P_iA_i$$
$$+\Delta_i(P_i) + \Psi_i(P_1,\dots,P_N) + (1+\mu_i)Q_i$$

2. If the LMI problem on the previous step is feasible then compute a static output feedback gain by the formula (16)

3. If the LMI with respect to variables S_i $(i \in \mathbb{N})$

$$(A_i - B_iF_iC_i)^{\mathrm{T}}S_i + S_i(A_i - B_iF_iC_i) + \Psi_i(S_1,\dots,S_N)$$
$$+\sum_{l=1}^{m}\gamma_{li}^2(A_{li} - B_{li}F_iC_i)^{\mathrm{T}}S_i(A_{li} - B_{li}F_iC_i) < \mathbf{0}$$

is feasible, then F_i $(i \in \mathbb{N})$ given by (16) is a ESMS gain.

B. Convex approximation II

Let for some parameter matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ $(i \in \mathbb{N})$ the following system of linear matrix inequalities with respect to variables $X_i = X_i^{\mathrm{T}} > \mathbf{0}$ and Y_i holds:

$$\begin{bmatrix} \Lambda_{11i} & \Lambda_{12i} & \Lambda_{13i} & \Lambda_{14i} \\ \Lambda_{12i}^{\mathrm{T}} & \Lambda_{22i} & \mathbf{0} & \mathbf{0} \\ \Lambda_{13i}^{\mathrm{T}} & \mathbf{0} & \Lambda_{33i} & \mathbf{0} \\ \Lambda_{14i}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \Lambda_{44i} \end{bmatrix} < \mathbf{0},\ i \in \mathbb{N}, \tag{17}$$

where

$$\Lambda_{11i} = [(A_iX_i - B_iY_iC_i)^{\mathrm{T}} + (A_iX_i - B_iY_iC_i) + \pi_{ii}X_i],$$
$$\Lambda_{12i} = [\pi_{i1}^{\frac{1}{2}}\Phi_{i1}^{\mathrm{T}}X_i \dots \pi_{ii-1}^{\frac{1}{2}}\Phi_{ii-1}^{\mathrm{T}}X_i\ \pi_{ii+1}^{\frac{1}{2}}\Phi_{ii+1}^{\mathrm{T}}X_i \dots \pi_{iN}^{\frac{1}{2}}\Phi_{iN}^{\mathrm{T}}X_i],$$
$$\Lambda_{13i} = [\gamma_{1i}(A_{1i}X_i - B_{1i}Y_iC_i)^{\mathrm{T}} \dots \gamma_{mi}(A_{mi}X_i - B_{mi}Y_iC_i)^{\mathrm{T}}],$$
$$\Lambda_{14i} = [X_iQ_i^{\frac{1}{2}}\ \ C_i^{\mathrm{T}}Y_i^{\mathrm{T}}],$$
$$\Lambda_{22i} = \mathrm{diag}[-X_1 \dots - X_{i-1}\ - X_{i+1} \dots - X_N]$$
$$\Lambda_{33i} = \mathrm{diag}[-X_i \dots - X_i],\ \ \Lambda_{44i} = \mathrm{diag}[-I_{n_x}\ - R_i^{-1}].$$

Following to (Crusius and Trofino, 1999) assume that there exists a decision variables Z_i $(i \in \mathbb{N})$ such that

$$C_iX_i = Z_iC_i \tag{18}$$

and suppose

$$F_i = Y_iZ_i^{-1}. \tag{19}$$

Denote $P_i = X_i^{-1}$ $(i \in \mathbb{N})$. Then taking into account (18), (19) and using Schur complement arguments rewrite (17) in the following form

$$(A_i - B_iF_iC_i)^{\mathrm{T}}P_i + P_i(A_i - B_iF_iC_i)$$
$$+\sum_{l=1}^{m}\gamma_{li}^2(A_{li} - B_{li}F_iC_i)^{\mathrm{T}}P_i(A_{li} - B_{li}F_iC_i)$$
$$+\Psi_i(P_1,\dots,P_N) + Q_i + (F_iC_i)^TR_iF_iC_i < \mathbf{0},\ \ i \in \mathbb{N}. \tag{20}$$

Because $P_i = P_i^{\mathrm{T}} > \mathbf{0}$ it follows from (20) that system (1)-(4) is ESMS and stabilizing gain is given by (19)

So we have the following result

Corollary 2: Let for some parameter matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ $(i \in \mathbb{N})$ the system of coupled linear matrix equations and inequalities (17), (18) with respect to variables X_i, Y_i and Z_i $(i \in \mathbb{N})$ is feasible. Then the control law (4) with the gain matrix F_i (19) provides ESMS of the system (1).

Based on these sufficient conditions it is easy to formulate the algorithm for obtaining of the stabilizing gain.

Algorithm 2:

1. Assign matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ $(i \in \mathbb{N})$, based on LQR reasons and solve the LMI/LME problem (17), (18) with respect to variables X_i, Y_i and Z_i $(i \in \mathbb{N})$.

2. If the LMI/LME problem on the previous previous step is feasible then compute the static output feedback stabilizing gain matrix F_i by the formula (19).

V. APPLICATION TO SIMULTANEOUS AND ROBUST STABILIZATION

A. Simultaneous stabilization

The particular case $F_i = F$, $\pi_{ij} \equiv 0$, $i,j \in \mathbb{N}$corresponds to the problem of simultaneous stabilization of the set of linear diffusion systems

$$dx(t) = [A_ix(t) + B_iu(t)] + \sum_{l=1}^{m}\gamma_{li}[A_{li}x(t) + B_{li}u(t)]dw_l(t), \tag{21}$$
$$y(t) = C_ix(t),\ t \geq 0,\ \ i \in \mathbb{N}$$

via output feedback with constant gain

$$u(t) = -Fy(t). \tag{22}$$

Corollary 3: Let for some scalar $\mu_i > 0$ and parameter matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ $(i \in \mathbb{N})$ the system of matrix Riccati equations

$$A_i^{\mathrm{T}}P_i + P_iA_i - [P_iB_i + \Theta_i(P_i)][R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}}] + (1+\mu_i)Q_i = \mathbf{0}. \tag{23}$$

has positive definite solution $P_i = P_i^{\mathrm{T}} > 0$ such that

$$\begin{bmatrix} \mu_iQ_i & L_i^{\mathrm{T}} \\ L_i & R_i + \Gamma_i(P_i) \end{bmatrix} > \mathbf{0},$$
$$(B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i)(\mathbf{1} - C_i^{+}C_i) = \mathbf{0}, \tag{24}$$
$$(R_i + \Gamma_i(P_i))^{-1}(B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i]C_i^{+}$$
$$= [R_{i+1} + \Gamma_{i+1}(P_{i+1})]^{-1}[B_{i+1}^{\mathrm{T}}P_{i+1} + \Theta_{i+1}(P_{i+1})^{\mathrm{T}} + L_{i+1}]C_{i+1}^{+},\ \ i \in \mathbb{N}$$

for some matrix L_i $(i \in \mathbb{N})$. Then the control law (22) with the gain matrix given by

$$F = [R_i + \Gamma_i(P_i)]^{-1}[B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i]C_i^{+} \tag{25}$$

for some $i \in \mathbb{N}$ provides ESMS of all the systems (21).

Based on that result and with the same methodology one gets the following algorithm to produce simultaneously stabilizing gains. Note that as the previous algorithm it is used the same conservative assumptions. Moreover, in order to have a unique feedback gain for all systems two additional assumptions in the form of the equality constraints are added.

Algorithm 3:

1. Assign matrix $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, and scalar $\mu_i > 0 \quad (i \in \mathbb{N})$ based on LQR reasons and solve the following LMI optimization problem with respect to variables $P_i = P_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ and L_i:

$$\mathrm{Tr}\sum_{j=1}^{N} P_j \longrightarrow \max$$

$$\begin{bmatrix} \Lambda(P_i) & P_iB_i + \Theta_i(P_i) \\ B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} & \Gamma_i(P_i) + R_i \end{bmatrix} \geq \mathbf{0},$$

$$\begin{bmatrix} \mu_iQ_i & L_i^{\mathrm{T}} \\ L_i & \Gamma_i(P_i) + R_i \end{bmatrix} > 0,$$

$$(B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i)(I - C_i^{+}C_i) = \mathbf{0},$$

$$(R_i + \Gamma_i(P_i)) = (R_{i+1} + \Gamma_{i+1}(P_{i+1})),$$

$$(B_i^{\mathrm{T}}P_i + \Theta_i(P_i)^{\mathrm{T}} + L_i)C_i^{+}$$
$$= (B_{i+1}^{\mathrm{T}}P_{i+1} + \Theta_{i+1}(P_{i+1})^{\mathrm{T}} + L_{i+1})C_{i+1}^{+}, \quad i \in \mathbb{N},$$

where $\Lambda(P_i) = A_i^{\mathrm{T}}P_i + P_iA_i + \Delta_i(P_i) + (1+\mu_i)Q_i$

2. If the LMI optimization problem on the previous step is feasible then compute a static output feedback gain by the formula (25)

3. If the LMI with respect to variables S_i $(i \in \mathbb{N})$

$$(A_i - B_iFC_i)^{\mathrm{T}}S_i + S_i(A_i - B_iFC_i)$$
$$+\sum_{l=1}^{m}\gamma_{li}^2(A_{li} - B_{li}FC_i)^{\mathrm{T}}S_i(A_{li} - B_{li}FC_i) < \mathbf{0}$$

is feasible, then the control law (22) with the gain matrix F given by formula (25) is simultaneously stabilizing one.

B. Application to robust stabilization problem

Assume now that the pairs of matrices $(A_i\ B_i)$ are vertices of a polytope defining an uncertain model in which the matrix C defining measurements is uncertainty independent and assume one seeks for a unique Lyapunov matrix $P_i = P$ $(i \in \mathbb{N})$. This case corresponds to the problem of quadratic stabilization via output feedback (22) of the linear system with polytopic uncertainty

$$\dot{x}(t) = \sum_{i=1}^{N}\xi_i(t)[A_ix(t) + B_iu(t)],\ y(t) = Cx(t), \qquad (26)$$

where $\xi(t) = \begin{pmatrix} \xi_1(t) & \dots & \xi_N(t) \end{pmatrix}$ belongs for all t to the simplex

$$\Xi = \left\{ \xi_i \geq 0\ ,\ \sum_{i=1}^{N}\xi_i = 1 \right\}.$$

Results of Theorem 1 apply and produce the following corollary.

Corollary 4: There exists a gain matrix F such that the uncertain system (26), (22) is quadratically stable if and only if there exist parameter matrices $Q_i = Q_i^{\mathrm{T}} > \mathbf{0}$, $R_i = R_i^{\mathrm{T}} > \mathbf{0}$ such that

$$FC = R_i^{-1}[B_i^{\mathrm{T}}P + L_i],\ i \in \mathbb{N},$$

where $P = P^T > 0$ and L_i $(i \in \mathcal{N})$ is a solution to the system of matrix inequalities

$$A_i^{\mathrm{T}}P + PA_i - PB_iR_i^{-1}B_i^{\mathrm{T}}P + Q_i + L_i^{\mathrm{T}}R_i^{-1}L_i \leq 0,\ i \in \mathbb{N}.$$

Based on this corollary and with the the same methodology as upper we can formulate the following robust stabilization algorithm. Its conservatism is analogous to the previous ones, only with the assumption that the C matrix is unique for all uncertainties.

Algorithm 4:

1. Assign scalar $\mu > 0$ matrices Q, R based on LQR reasons on the vertices of the polytope and solve the following LMI/LME problem with respect to variables P, L_i

$$\mathrm{Tr}P \longrightarrow \max$$

$$\begin{bmatrix} A_i^{\mathrm{T}}P + PA_i + (1+\mu)Q & PB_i \\ B_i^{\mathrm{T}}P & R \end{bmatrix} \geq \mathbf{0},$$

$$\begin{bmatrix} \mu Q & L_i^{\mathrm{T}} \\ L_i & +R \end{bmatrix} > \mathbf{0}, \qquad (27)$$

$$(B_i^{\mathrm{T}}P + L_i)(\mathbf{1} - C^{+}C) = \mathbf{0}, (B_i^{\mathrm{T}}P + L_i) \qquad (28)$$

$$= (B_{i+1}^{\mathrm{T}}P + L_{i+1}),\ \ i \in \mathbb{N}, \qquad (29)$$

2. If the LMI with respect to variable S

$$(A_i - B_iFC)^{\mathrm{T}}S + S(A_i - B_iFC)$$
$$+\sum_{l=1}^{m}\gamma_{li}^2(A_{li} - B_{li}FC)^{\mathrm{T}}S(A_{li} - B_{li}FC) < \mathbf{0}$$

is feasible, then the control law (22) with the gain matrix F given by formula

$$F = R^{-1}[B_i^{\mathrm{T}}P_i + L_i]C^{+}$$

for some $i \in \mathbb{N}$ is quadratically stabilizing one.

It is easy to see that Algorithm 2 and Algorithm 3 can be also applied with corresponding conservative assumptions.

VI. STOCHASTIC PASSIVITY AND PASSIFICATION

Stochastic passivity and dissipativity properties have been studied in [5], [18], [27] and references therein. We consider here a particular case of stochastic exponential dissipativity, see [18]. It is stochastic counterpart of G-passivity [2]. Define for the system (1) some input w and the output $z$of the same dimensions by the formula

$$z(t) = G(r(t))y(t) + D(r(t)w(t), \qquad (30)$$

where G and D are matrices of compatible dimension.

Definition 2: System (1) is said to be stochastically G-passive with respect to input w and output z if there exists nonnegative scalar function $V(x,i)$ $(x \in \mathbb{R}^{n_x},\ i \in \mathbb{N})$ and scalar function $\rho(x,i) > 0$ for $x \neq 0$ $(x \in \mathbb{R}^{n_x},\ i \in \mathbb{N})$ such that

$$\mathrm{E}_{xi}V(x(t),r(t)) \leq V(x,i) + \mathrm{E}_{xi}\int_0^t [w^{\mathrm{T}}(s)z(s)$$
$$-\rho(x(s),r(s))]ds, \qquad (31)$$

for each solution of the system (1) with deterministic initial conditions $x(0) = x,\ r(0) = i$, where E_{xi} is expectation operator with $x(0) = x,\ r(0) = i$.

The stochastic passification problem is to find the triple of matrices $(F_i, G_i, D_i)\ (i \in \mathbb{N})$ such that the system (1) with reference input w and with static output feedback

$$u(t) = w(t) - F(r(t))y(t) \tag{32}$$

is exponentially stable in the mean square and stochastically G-passive with respect to input w and output z.Consider $V(x,i) = x^{\mathrm{T}}H_ix,\quad H_i = H_i^{\mathrm{T}} > \mathbf{0}\ (x \in \mathbb{R}^{n_x}, i \in \mathbb{N})$ as a candidate storage function and let $\mu(x,i) = x^{\mathrm{T}}W_ix,\quad W_i^{\mathrm{T}} = W_i^{\mathrm{T}} > 0\ (x \in \mathbb{R}^{n_x}, i \in \mathbb{N})$, then according to [18] and (31) the stochastic G-passivity conditions with respect to input (32) and output (30) can be written as

$$\begin{bmatrix} (\Omega(H_1,\ldots,H_N) & H_iB_i - (G_iC_i)^{\mathrm{T}} \\ B_i^{\mathrm{T}}H_i - G_iC_i & -D_i - D_i^{\mathrm{T}} \end{bmatrix} \leq \mathbf{0},\ i \in \mathbb{N}, \tag{33}$$

where $A_{ci} = A_i - B_iF_iC_i,\quad A_{cli} = A_{li} - B_{li}F_iC_i\quad \Omega(H_1,\ldots,H_N) = A_{ci}^{\mathrm{T}}H_i + H_iA_{ci} + W_i + \Psi_i(H_1,\ldots,H_N) + \sum_{l=1}^{m}\gamma_{li}^2A_{cli}^{\mathrm{T}}H_iA_{cli}$.

If matrix $F_i\ (i \in \mathbb{N})$ is known then these bilinear matrix inequalities will be LMIs with respect to matrix G and the passification problem can be solved in the following way. Find the matrix $F_i\ (i \in \mathbb{N})$ using Algorithm 1 or 2. Then for obtained matrix $F_i\ (i \in \mathbb{N})$ find matrices $G_i\ (i \in \mathbb{N})$ and $D_i\ (i \in \mathbb{N})$ as a solution of LMI (33).

Note that given a ESMS gain $F_i\ (i \in \mathbb{N})$, any matrix $G_i\ (i \in \mathbb{N})$ is solution to the problem if one takes $D_i\ (i \in \mathbb{N})$ positive definite with sufficiently large eigenvalues. But recall that $D_i\ (i \in \mathbb{N})$ is some parallel feedthrough gain. In practice one would expect it to be zero. Hence it is required to have $D_i\ (i \in \mathbb{N})$ as close to zero as possible and hence the LMIs should be solved along with some minimization of the norm of $D_i\ (i \in \mathbb{N})$ for example by performing

$$\sum_{i=1}^{N}\mathrm{Trace}(D_i) \longrightarrow \min\,, \quad D_i + D_i^{\mathrm{T}} \geq \mathbf{0}\ i \in \mathbb{N}. \tag{34}$$

It one gets at the optimum $D_i = \mathbf{0}\ (i \in \mathbb{N})$ then passivity is demonstrated with respect to the output $z(t) = G(r(t))y(t)$. Finally the following algorithm is proposed to find the stochastically G-passive output.

Algorithm 5:

1. Solve the output stabilization problem using Algorithm 1 or 2 and find the gain matrix $F_i,\ i \in \mathbb{N}$
2. Solve the LMI optimization problem (34), (33) and find the matrices G_i and $D_i,\ \ i \in \mathbb{N}$.

VII. NUMERICAL EXAMPLE

Consider the linearized model of L-1011 aircraft in cruise flight conditions [12]. For this model

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & -0.154 & -0.0042 & 1.54 & 0 \\ 0 & 0.249 & -1 & -5.2 & 0 \\ 0.0386 & -0.996 & -0.0003 & -0.117 & 0 \\ 0 & 0.5 & 0 & 0 & -0.5 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & -0.744 & 0.337 & 0.02 & 0 \\ 0 & -0.032 & -1.12 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

The matrix C has two mode

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The random switching between these modes is governed by the Markov chain $r(t)$ with state space $\mathbb{N} = 1, 2$ and with infinitesimal matrix

$$\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & -0.5 \end{bmatrix}.$$

The problem is to stabilize the system

$$\dot{x}(t) = Ax(t) + Bu(t),\ y(t) = C(r(t))x(t), \tag{35}$$

by means of switching static output feedback control law (4) and obtain the matrices of some stochastically passive output (30).

Algorithm 1 is applied with $Q_i = I_{n_x},\ R_i = I_{n_u}$ and $\mu_i = 1$. LMIs of step 1 are found feasible and step 2 gives the following switching static output-feedback gain of stabilizing controller (4)

$$F_1 = \begin{bmatrix} -0.8576 & 0.0581 & 1.5320 & 0.0509 \\ -0.2871 & -1.0940 & 2.2458 & -1.1097 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -0.6020 & 0.4097 & 0 & 0 \\ 0.1556 & -0.3313 & 0 & 0 \end{bmatrix}.$$

For that values of F_i the Algorithm 5 gives the following values for gains of the passified output

$$G = \begin{bmatrix} -1.8327 & 0.8863 & 0.0633 & -0.1653 \\ 0.2317 & -2.2602 & 0.0026 & -0.0562 \end{bmatrix}$$

$$D = 1.0\mathrm{e} - 009 \begin{bmatrix} 0.2745 & -0.0005 \\ -0.0005 & 0.3482 \end{bmatrix}$$

Figure 1 gives the times histories of the passified output $z = Gy + Dw$ for step inputs on the reference w plotted for two modes.

All LMI/LME programming was done using YALMIP parser [13] and solved with SeDuMi solver [25]. Computation time was about two second for Algorithm 5 with using Algorithm 1 for finding output feedback gain.

VIII. CONCLUSIONS

LQR type paramerization of static output feedback gains for diffusion systems with Markovian switching is proposed. This parametrization gives more complicated non-convex relations than original Lyapunov like inequalities, but it turns out that convex approximation technique can be effectively used to these relation to obtain LMI-based algorithm for

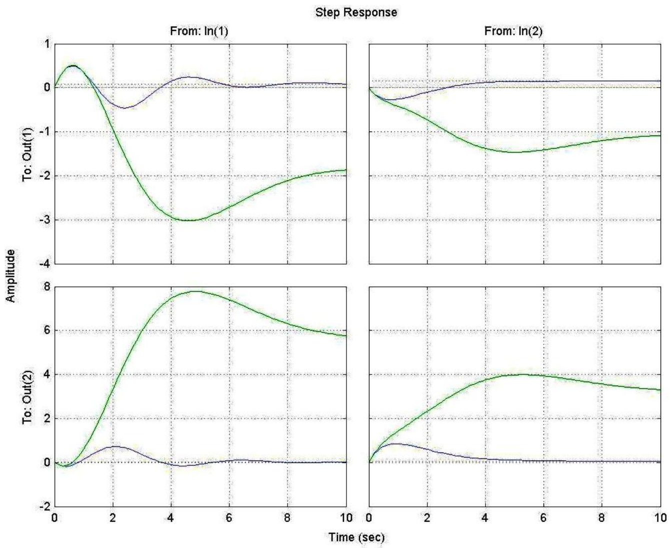

Fig. 1. Time histories of the passified output

computing of stabilizing gain. This result then applied to simultaneous and robust stabilization and robust passification problems. An illustrative numerical example is given. The results are conservative because additional convexifying restrictions. The evaluation of the degree of conservatism is an interesting open problem.

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