Disturbance Decoupling Problems with Quadratic Stability for Switched Linear Systems via State Feedback

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***Abstract*— In this paper disturbance decoupling problems without stability and with quadratic stability for switched linear systems are formulated in the framework of the so-called geometric approach. Firstly, necessary and sufficient conditions for the problem without stability to be solvable are given. Secondly, sufficient conditions for the problem with quadratic stability to be solvable are given. Further, for switched linear systems composed of two subsystems necessary and sufficient conditions for the problem with quadratic stability to be solvable are also investigated. Finally, an illustrative example is shown.**

I. INTRODUCTION.

In the framework of the so-called geometric approach, decoupling problems and disturbance decoupling problems have been studied for time invariant linear systems by using the concepts of some invariant subspaces (e.g.,[1],[19]). After that some invariant subspaces have been extended to various types of systems (e.g., infinite-dimensional linear systems[3],[4],[20], discrete-time periodic systems[7], non-linear systems[9],[11], uncertain linear systems[2],[6], [12]-[16] etc) in order to study the same problems. Among those studies simultaneous invariant subspaces have been used to study a family of linear systems and / or uncertain linear systems, and some useful results have been obtained.

On the other hand, the so-called switched system is composed as the family of subsystems with switching rule which concerns with various environmental factors and different controllers, and the stability problems for various types of switched systems have been studied (e.g., [5],[8],[10],[17],[18]). As an interesting result sufficient conditions for the switched linear systems by state feedback to be quadratically stabilizable were given[5],[8],[18]. For the case that the number of subsystems is two, it was shown that the conditions are also necessary by Feron[5]. However, disturbance decoupling problems for switched systems have not been studied in the framework of the geometric approach.

In this paper disturbance decoupling problems without stability and with quadratic stability for switched linear systems are formulated, and solvability conditions for the problems are proved by using the concepts of simultaneous $\{(A_i, B_i); i = 1, 2, \cdots, N\}$-invariant subspaces and the results of quadratic stabilizability for switched systems.

In Section II some fundamental results which will be needed in the main section are discussed. In Section III problems formulation and the main results are given. Further, an illustrative example is also investigated in Section IV. Finally, conclusions are given in Section V.

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II. PRELIMINARIES.

At first, we give some notations. For a linear map $A : \mathcal{X} \to \mathcal{Y}$ from a vector space $\mathcal{X}$ into a vector space $\mathcal{Y}$ and a subspace φ of $\mathcal{Y}$ the image, the kernel, the dimension and the inverse image are denoted by Im(A), Ker(A), dim(φ) and $A^{-1}\varphi := \{x \in \mathcal{X} \mid Ax \in \varphi\}$, respectively. Further, for a linear map A from a vector space $\mathcal{X} := \Re^n$ into itself and a subspace φ of $\mathcal{X}$ define $< A \mid \varphi >:= \varphi + A\varphi + \cdots + A^{n-1}\varphi$.

Next, consider the following continuous-time switched linear system :

$$\Sigma_\sigma : \dot{x}(t) = A_{\sigma(x,t)}x(t) + B_{\sigma(x,t)}u(t),\ x(0) = x_0,$$

where $x(t) \in \mathcal{X} := \Re^n$ is the state, $u(t) \in \mathcal{U} := \Re^m$ is the input, $\sigma(x,t) : \Re^n \times \Re^+ \to \{1, 2, \cdots, N\}$ is a switched rule which depends on the state x and time t, and $\Re^+$ is the set of non-negative real numbers. We shall often use the shorter notation σ to be denote the switching rule.

Then, the above switched system Σ_σ is composed as the family of continuous-time subsystems

$$\Sigma_i : \dot{x}(t) = A_i x(t) + B_i u(t), \ \ i = 1, 2, \cdots, N,$$

where $N\ (\geq 2)$ is the number of subsystems, $A_i :\ \mathcal{X} \to \mathcal{X}$, $B_i :\ \mathcal{U} \to \mathcal{X}\ \ (i = 1, \cdots, N)$ are linear maps. For a set of N subspaces $\mathcal{V}$ of $\mathcal{X}$ the external direct sum $\mathcal{V} \oplus \mathcal{V} \oplus \cdots \oplus \mathcal{V}$ is denoted by $\mathcal{V}^{\oplus N}$. Further, let $A_1 \oplus A_2 \oplus \cdots \oplus A_N$ be a linear map from $\mathcal{X}$ into $\mathcal{X}^{\oplus N}$ given by

$$\mathcal{X} \ni x \ \mapsto (A_1 x \oplus A_2 x \oplus \cdots \oplus A_N x) \in \mathcal{X}^{\oplus N}.$$

The following definition plays an important role in this study.

Definition 2.1: Let $\mathcal{V}$ and Ω be subspaces of $\mathcal{X}$.

(i) $\mathcal{V}$ is said to be *simultaneous* $\{(A_i, B_i); i = 1, \cdots, N\}$-*invariant* if

$$(A_1 \oplus A_2 \oplus \cdots \oplus A_N)\mathcal{V} \subset \mathcal{V}^{\oplus N} + (\mathrm{Im}B_1 \oplus\ \mathrm{Im}B_2 \oplus \cdots \oplus\ \mathrm{Im}B_N).$$

In this case, we define

$$\mathcal{V}_{\mathbf{sim}}(\Omega) := \{\mathcal{V}\ (\ \subset \Omega) \mid \mathcal{V} \text{ satisfies the above condition } \}.$$

(ii) $\mathcal{V}$ is said to be *simultaneous* $\{(A_i, B_i); i = 1, \cdots, N\}$-*invariant with a common input* if

$$(A_1 \oplus A_2 \oplus \cdots \oplus A_N)\mathcal{V} \subset \mathcal{V}^{\oplus N} + \mathrm{Im}(B_1 \oplus B_2 \oplus \cdots \oplus B_N).$$

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In this case, we define

$$\mathcal{V}^{\mathbf{com}}_{\mathbf{sim}}(\Omega) := \{\mathcal{V}\ (\subset \Omega) \mid \mathcal{V} \text{ satisfies the above condition}\}. ■$$

Now, if we apply a state feedback of the form $u(t) = F_\sigma x(t)$, where $F_\sigma : \mathcal{X} \to \mathcal{U}$ to the switched system Σ_σ, then we have the closed-loop switched system

$$\Sigma_\sigma^{F_\sigma} : \dot{x}(t) = (A_\sigma + B_\sigma F_\sigma)x(t).$$

If we apply a common state feedback of the form $u(t) = Fx(t)$, where $F : \mathcal{X} \to \mathcal{U}$ to the switched system Σ_σ, then we have

$$\Sigma_\sigma^{F} : \dot{x}(t) = (A_\sigma + B_\sigma F)x(t).$$

For the families of subsystems $\Sigma_i^{F_i}$ and Σ_i^{F} the following lemma holds, and state feedback gain for a simultaneous invariant subspace can be easily designed (e.g.,[6],[16]).

Lemma 2.2:

(i) $\mathcal{V}$ is simultaneous $\{(A_i, B_i); i = 1, \cdots, N\}$-invariant if and only if there exist $F_i : \mathcal{X} \to \mathcal{U}$ such that

$$(A_i + B_i F_i)\mathcal{V} \subset \mathcal{V} \quad \text{for all} \quad i = 1, \cdots, N.$$

In this case, we define

$$\mathcal{F}((A_i, B_i); \mathcal{V}) := \{F_i : \mathcal{X} \to \mathcal{U} \mid (A_i + B_i F_i)\mathcal{V} \subset \mathcal{V}\}$$

for all $i = 1, \cdots, N$.

(ii) $\mathcal{V}$ is simultaneous $\{(A_i, B_i); i = 1, \cdots, N\}$-invariant with a common input if and only if there exists an $F : \mathcal{X} \to \mathcal{U}$ such that

$$(A_i + B_i F)\mathcal{V} \subset \mathcal{V} \quad \text{for all} \quad i = 1, \cdots, N.$$

In this case, we define

$$\mathcal{F}^{com}(\mathcal{V}) := \{F : \mathcal{X} \to \mathcal{U} \mid (A_i + B_i F)\mathcal{V} \subset \mathcal{V} \quad \text{for all} \quad i = 1, \cdots, N\}. ■$$

The following remark can be easily obtained.

Remark 2.3:

(i) Lemma2.2(ii) implies Lemma2.2(i).

(ii) $\mathcal{F}^{com}(\mathcal{V}) = \bigcap_{i=1}^{N} \mathcal{F}\ ((A_i, B_i); \mathcal{V})$. ■

The following results are needed to check the solvability conditions of the main results (e.g.,[16]).

Lemma 2.4:

(i) The class $\mathcal{V}_{\mathbf{sim}}(\Omega)$ has the unique maximal element $\mathcal{V}^*_{sim}(\Omega)$ and it can be computed as the following sequence.

(Step 1) $\mathcal{V}_{(0)} := \Omega$,

(Step 2) $\mathcal{V}_{(k)} := \Omega \cap (A_1 \oplus A_2 \oplus \cdots \oplus A_N)^{-1}\{\mathcal{V}^{\oplus N}_{(k-1)} + (\mathrm{Im}B_1 \oplus \mathrm{Im}B_2 \oplus \cdots \oplus \mathrm{Im}B_N)\}$ $(k = 1, 2, \cdots)$,

(Step 3) $\mathcal{V}^*_{sim} := \mathcal{V}_{(\dim(\Omega))}$.

(ii) The class $\mathcal{V}^{\mathbf{com}}_{\mathbf{sim}}(\Omega)$ has the unique maximal element $\mathcal{V}^{com^*}_{sim}(\Omega)$ and it can be computed as the following sequence.

(Step 1) $\mathcal{V}_{(0)} := \Omega$,

(Step 2) $\mathcal{V}_{(k)} := \Omega \cap (A_1 \oplus A_2 \oplus \cdots \oplus A_N)^{-1}\{\mathcal{V}^{\oplus N}_{(k-1)} + \mathrm{Im}(B_1 \oplus B_2 \oplus \cdots \oplus B_N)\}$ $(k = 1, 2, \cdots)$,

(Step 3) $\mathcal{V}^{com^*}_{sim} := \mathcal{V}_{(\dim(\Omega))}$. ■

Now, we give the definition of quadratic stabilizability via state feedback for the switched system Σ_σ.

Definition 2.5: The switched linear system Σ_σ is quadratically stabilizable via state feedback if there exist a switched rule $\sigma(x, t)$, a state feedback $F_\sigma : \mathcal{X} \to \mathcal{U}$, a positive number $\epsilon\ (> 0)$ and a Lyapunov function of the form $V(x) = x^{\mathrm{T}}Px$, where P is a positive-definite matrix such that

$$\frac{d}{dt}V(x) < -\epsilon x^{\mathrm{T}}x$$

for all trajectories x of the closed-loop switched system $\Sigma_\sigma^{F_\sigma}$. ■

The following lemma is a well known result for quadratic stabilizability of switched linear system (e.g., [5], [8]).

Lemma 2.6: The switched system Σ_σ is quadratically stabilizable via state feedback if there exist state feedback $F_i : \mathcal{X} \to \mathcal{U}$ and $\lambda_i \in [0, 1]$ $(i = 1, \cdots, N)$ satisfying $\sum_{i=1}^{N} \lambda_i = 1$ such that

$$\sum_{i=1}^{N} \lambda_i (A_i + B_i F_i) \text{ is Hurwitz stable.}$$

Further, for the case of $N = 2$, the condition is also necessary. ■

III. Problem Formulation and Main Results.

Consider the following switched system Σ_σ of Section II with disturbance ξ described as

$$\Sigma_\sigma^{\xi} : \begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + E_\sigma \xi(t),\ x(0) = x_0 \\ y(t) = C_\sigma x(t), \end{cases}$$

where $x(t) \in \mathcal{X} := \Re^n$ is the state, $u(t) \in \mathcal{U} := \Re^m$ is the input, $y(t) \in \mathcal{Y} := \Re^\ell$ is the output and $\xi(t) \in \Xi := \Re^\eta$ is the disturbance. If we apply a state feedback of the form $u(t) = F_\sigma x(t)$ to the switched system Σ_σ^{ξ}, then the following closed-loop system can be obtained.

$$\Sigma_\sigma^{\xi, F_\sigma} : \begin{cases} \dot{x}(t) = (A_\sigma + B_\sigma F_\sigma)x(t) + E_\sigma \xi(t),\ x(0) = x_0 \\ y(t) = C_\sigma x(t). \end{cases}$$

Now, we have the following closed-loop subsystems $\{\Sigma_i^{\xi, F_i}; i = 1, 2, \cdots, N\}$:

$$\Sigma_i^{\xi, F_i} : \begin{cases} \dot{x}(t) = (A_i + B_i F_i)x(t) + E_i \xi(t) \\ y(t) = C_i x(t). \end{cases}$$

Suppose that the subsystems are activated by the switched rule as follows.

$$\Sigma_{i_1}^{\xi, F_{i_1}} \to \Sigma_{i_2}^{\xi, F_{i_2}} \to \Sigma_{i_3}^{\xi, F_{i_3}} \to \cdots, \tag{1}$$

where $i_1, i_2, i_3, \cdots \in \{1, 2, \cdots, N\}$.

Since a subsystem $\Sigma_{i_1}^{\xi,F_{i_1}}$ is firstly activated by (1), the subspace generated by disturbances through $\mathrm{Im}E_{i_1}$ is

$$\mathcal{V}_{i_1}^{F_{i_1}} :=< A_{i_1} + B_{i_1}F_{i_1} \mid \mathrm{Im}E_{i_1} >$$
$$= \{\int_0^t e^{(A_{i_1}+B_{i_1}F_{i_1})(t-\tau)}E_{i_1}\xi(\tau)d\tau \mid \forall\xi \in \Xi,\ \forall t\}.$$

If the subsystem is changed from $\Sigma_{i_1}^{\xi,F_{i_1}}$ to $\Sigma_{i_2}^{\xi,F_{i_2}}$ by the switched rule (1), then the subspace generated by disturbances through $\mathcal{V}_{i_1}^{F_{i_1}}$ and $\mathrm{Im}E_{i_2}$ is

$$\mathcal{V}_{i_2}^{F_{i_2}} :=< A_{i_2} + B_{i_2}F_{i_2} \mid \mathcal{V}_{i_1}^{F_{i_1}} + \mathrm{Im}E_{i_2} > .$$

Similarly, we have the following subspaces.

$$\mathcal{V}_{i_j}^{F_{i_j}} :=< A_{i_j}+B_{i_j}F_{i_j} \mid \mathcal{V}_{i_{(j-1)}}^{F_{i_{(j-1)}}}+\mathrm{Im}E_{i_j} > \quad (j = 2, 3, \cdots).$$

From the construction of subspaces $\mathcal{V}_{i_j}^{F_{i_j}}$ $(j = 1, 2, \cdots)$ the following properties hold.

$$\mathrm{Im}E_{i_1} \subset \mathcal{V}_{i_1}^{F_{i_1}} \subset (\mathrm{Im}E_{i_2} + \mathcal{V}_{i_1}^{F_{i_1}}) \subset \mathcal{V}_{i_2}^{F_{i_2}} \subset \cdots$$
$$\subset \mathcal{V}_{i_{(j-1)}}^{F_{i_{(j-1)}}} \subset \mathcal{V}_{i_j}^{F_{i_j}} \subset \cdots .$$

Since $\mathcal{X}$ is finite-dimensional, there exists a finite number μ such that

$$\varphi := \mathcal{V}_{i_\mu}^{F_{i_\mu}} = \mathcal{V}_{i_j}^{F_{i_j}} \text{ for all } j \geq \mu.$$

Thus, the subspace φ contains $\sum_{i=1}^{N}\mathrm{Im}E_i$ and is $(A_i + B_iF_i)$-invariant for all $i = 1, 2, \cdots, N$, that is, φ is simultaneous $\{(A_i, B_i); i = 1, \cdots, N\}$-invariant and is a subspace generated by disturbances for the switched system $\Sigma_\sigma^{\xi,F_\sigma}$.

Then, our disturbance decoupling problem for the switched linear system Σ_σ^ξ is to find state feedback $F_i : \mathcal{X} \to \mathcal{U}$ $(i = 1, 2, \cdots, N)$ such that the output to be controlled is not affected by disturbance $\xi(t)$. To achieve this control requirement we must solve the following problem.

Disturbance Decoupling Problem (DDP)

Given A_i, B_i, C_i, E_i for $i = 1, \cdots, N$ for the switched linear system Σ_σ^ξ, find (if possible) state feedback $F_i : \mathcal{X} \to \mathcal{U}$ $(i = 1, 2, \cdots, N)$ such that

$$\varphi \subset \bigcap_{i=1}^{N}\mathrm{Ker}C_i =: \mathit{\Lambda}$$

for the switched rule (1). ■

The following theorem is the first main result.

Theorem 3.1: DDP is solvable if and only if

$$\sum_{i=1}^{N}\mathrm{Im}E_i \subset \varphi_{sim}^*,$$

where φ_{sim}^* is the maximal element of $\mathcal{V}_{\mathbf{sim}}(\mathit{\Lambda})$ which is computed from Lemma 2.4(i). ■

The following corollary is the result of the case used a common state feedback.

Corollary 3.2: DDP with a common state feedback is solvable if and only if

$$\sum_{i=1}^{N}\mathrm{Im}E_i \subset \varphi_{sim}^{com^*},$$

where $\varphi_{sim}^{com^*}$ is the maximal element of $\mathcal{V}_{\mathbf{sim}}^{\mathbf{com}}(\mathit{\Lambda})$ which is computed from Lemma 2.4(ii). ■

Next, disturbance decoupling problem with quadratic stability via state feedback for switched system is formulated as follows.

Disturbance Decoupling Problem with Quadratic Stability(DDPQS)

Given A_i, B_i, C_i, E_i for $i = 1, 2, \cdots, N$ for the switched linear system Σ_σ^ξ, find (if possible) state feedback $F_i : \mathcal{X} \to \mathcal{U}$ $(i = 1, 2, \cdots, N)$ and a switched rule $\sigma(x, t)$ such that

$$\varphi \subset \mathit{\Lambda} \quad \text{and} \quad \Sigma_\sigma^{\xi,F_\sigma} \text{ is quadratically stable}$$

for the switched rule (1). ■

The following theorem is the second main result.

Theorem 3.3: DDPQS is solvable if there exist state feedback $F_i \in \mathcal{F}\ ((A_i, B_i); \varphi_{sim}^*)$ $(i = 1, 2, \cdots, N)$ and $\lambda_i \in [0, 1]$ $(i = 1, \cdots, N)$ satisfying $\sum_{i=1}^{N}\lambda_i = 1$ such that

(i) $\sum_{i=1}^{N}\mathrm{Im}E_i \subset \varphi_{sim}^*$ ($\Leftrightarrow$ DDP is solvable) and

(ii) $\sum_{i=1}^{N}\lambda_i(A_i + B_iF_i)$ is Hurwitz stable,

where φ_{sim}^* is the maximal element of $\mathcal{V}_{\mathbf{sim}}(\mathit{\Lambda})$. ■

The following corollary is the result of the case used a common state feedback.

Corollary 3.4: DDPQS with a common state feedback is solvable if there exist a state feedback $F \in \mathcal{F}^{com}(\varphi_{sim}^{com^*})$ and $\lambda_i \in [0, 1]$ $(i = 1, \cdots, N)$ satisfying $\sum_{i=1}^{N}\lambda_i = 1$ such that

(i) $\sum_{i=1}^{N}\mathrm{Im}E_i \subset \varphi_{sim}^{com^*}$

($\Leftrightarrow$ DDP with a common state feedback is solvable) and

(ii) $\sum_{i=1}^{N}\lambda_i(A_i + B_iF)$ is Hurwitz stable,

where $\varphi_{sim}^{com^*}$ is the maximal element of $\mathcal{V}_{\mathbf{sim}}^{\mathbf{com}}(\mathit{\Lambda})$. ■

The following theorem gives necessary and sufficient conditions for DDPQS to be solvable in the case of two subsystems.

Theorem 3.5: Suppose that $N = 2$. Then, DDPQS is solvable if and only if there exist state feedback $F_i \in$

$\mathcal{F}((A_i, B_i)\ ;\varphi^*_{sim})$ $(i = 1, 2)$ and $\lambda \in [0, 1]$ such that

(i) $(\mathrm{Im}E_1 + \mathrm{Im}E_2) \subset \varphi^*_{sim}$ ($\Leftrightarrow$ DDP is solvable) and

(ii) $\lambda(A_1 + B_1F_1) + (1-\lambda)(A_2 + B_2F_2)$ is Hurwitz stable, where φ^*_{sim} is the maximal element of $\mathcal{V}_{\mathbf{sim}}(\Lambda)$. ■

Similarly, we have the following corollary.

Corollary 3.6: Suppose that $N = 2$. Then, DDPQS with a common state feedback is solvable if and only if there exist a state feedback $F \in \mathcal{F}^{com}(\varphi^{com^*}_{sim})$ and $\lambda \in [0, 1]$ such that

(i) $(\mathrm{Im}E_1 + \mathrm{Im}E_2) \subset \varphi^{com^*}_{sim}$

($\Leftrightarrow$ DDP with a common state feedback is solvable) and

(ii) $\lambda(A_1+B_1F)+(1-\lambda)(A_2+B_2F)$ is Hurwitz stable,

where $\varphi^{com^*}_{sim}$ is the maximal element of $\mathcal{V}^{\mathbf{com}}_{\mathbf{sim}}(\Lambda)$. ■

IV. An Illustrative Example.

Consider the following two-dimensional switched linear system :

$$\Sigma^{\xi}_{\sigma} : \begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) + E_\sigma \xi(t),\ x(0) = x_0 \\ y(t) = C_\sigma x(t), \end{cases}$$

which is composed by two subsystems

$$\Sigma^{\xi}_{i}(i = 1, 2) : \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) + E_i \xi(t), \\ y(t) = C_i x(t), \end{cases}$$

where

$$A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, E_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2 & 1 \\ -1 & -\frac{3}{2} \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ and}$$

$$E_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Now, the maximal element $\varphi^{com^*}_{sim} := \mathrm{Im}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ of $\mathcal{V}^{\mathbf{com}}_{\mathbf{sim}}(\Lambda)$ can be computed from Lemma 2.4(ii), where $\Lambda = \mathrm{Ker}C_1 \cap \mathrm{Ker}C_2$. Then, there exists a state feedback $u(t) = \underbrace{\begin{bmatrix} f_1 & -1 \end{bmatrix}}_{=:F} x(t)$ where f_1 is an arbitrary element that $\varphi^{com^*}_{sim} := \mathrm{Im}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $(A_i + B_iF)$-invariant for all $i = 1, 2$, that is, $F \in \mathcal{F}^{com}(\varphi^{com^*}_{sim})$ and $\varphi^{com^*}_{sim} \subset \Lambda$.

Since (ImE_1+ImE_2)=Im$\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \varphi^{com^*}_{sim}$, it follows from Corollary 3.2 that DDP with a common state feedback is solvable.

Next, if we choose a state feedback gain $F^* = \begin{bmatrix} -2 & -1 \end{bmatrix}$ and $\lambda = \frac{1}{4} \in [0, 1]$, then the eigenvalues of matrix $\lambda(A_1 + B_1F^*) + (1-\lambda)(A_2 + B_2F^*)$ are $-\frac{1}{8}$ and $-\frac{1}{4}$ which implies $\frac{1}{4}(A_1+B_1F^*)+\frac{3}{4}(A_2+B_2F^*)$ is Hurwitz stable.

Thus, it follows from Corollary 3.6 that DDPQS with a common state feedback is solvable.

V. Conclusions.

In this paper disturbance decoupling problems without stability and with quadratic stability by two types of state feedback for switched linear systems were formulated in the framework of the so-called geometric approach. Firstly, necessary and sufficient conditions for the problem without stability to be solvable were given. Secondly, sufficient conditions for the problem with quadratic stability to be solvable were also given. In this case if the number of subsystems is two, then it was also shown that the conditions are necessary. Finally, an illustrative example was given.

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