Constrained control design – a simulation-based scenario approach

Maria Prandini and Marco C. Campi

***Abstract*— This paper deals with constrained control design for linear systems affected by stochastic disturbances. The goal is to optimize the control performance while guaranteeing that the constraints are satisfied for most of the disturbance realizations, that is with probability $1-\epsilon$. In mathematical terms, this amounts to solve a "chance-constrained" optimization program and we introduce here a randomized approach to this problem that builds on certain recent results in robust convex optimization.**

I. INTRODUCTION

This paper deals with constrained control design for linear systems affected by stochastic disturbances. The goal is to optimize the control performance over a finite time-horizon, while guaranteeing that the input/output signals satisfy given saturation/safety limits.

Stochastic disturbances normally have noncompact distribution support, so that guaranteeing the satisfaction of the limits for every noise realization is impractical, or even impossible. In order to avoid overconservatism, a chance-constrained approach is adopted where violation of constraints is admitted up to a predefined (small) probability level ϵ, [1], [2], [3].

By adopting a Youla parameterization of the controller, the chance-constrained design problem is reduced to a convex optimization program which involves an infinite number of disturbance realizations. The good news is that a feasible solution to this program can be computed by simulating finitely many, say N, disturbance realizations, the so-called 'scenario realizations', generated at random. The link to the initial problem with infinite realizations is established based on certain powerful results recently proved in connection with convex optimization through randomization ([4], [5], [6], [7]): if N is appropriately chosen, then the obtained randomized solution is guaranteed to be chance-constrained feasible, that is the saturation/safety limits are guaranteed for unseen disturbance realizations with probability $1-\epsilon$.

The proposed design method is named *simulation-based scenario (SBS)* approach after the idea of inspecting the control system behavior over situations studied through simulation.

Other methodologies for solving control design problems for linear systems affected by disturbances and subject to constraints are present in the literature of receding horizon and model predictive control, [8], [9], [10], [11]. Differently from what we propose here, no structure is imposed to the feedback controller in these papers and the design is carried out by directly optimizing over the control input samples in a time horizon of interest. The resulting feedback controller suffers from the problem to be difficult to implement, but it secures high performance under certain hypotheses. Moreover, applicability of standard methods in receding horizon and model predictive control requires that uncertainty is quite structured and, typically, with bounded support, a limitation which is overcome by the SBS approach proposed here.

The rest of the paper is organized as follows. In Section II, the SBS approach is described with reference to a tracking control problem. In Section III, an example of control design for an active suspension system is presented. Some concluding remarks are drawn in Section IV.

II. DESCRIPTION OF THE SBS APPROACH TO CONSTRAINED CONTROL DESIGN

Consider a discrete-time stable plant with scalar input and scalar output, $u(t)$ and $y(t)$, described by a transfer function $P(z)$ and affected by some additive disturbance $d(t)$:

$$y(t) = P(z)u(t) + d(t).$$

Suppose that a reference $y^\circ(t)$ is assigned over a time horizon of interest, say $[1,T]$, and the goal is to make $y(t)$ track $y^\circ(t)$ as close as possible, despite the presence of the stochastic disturbance $d(t)$. The tracking performance is evaluated through the 2-norm

$$\sum_{t=1}^{T}[y(t)-y^\circ(t)]^2.$$

Moreover, boundedness constraints on $u(t)$ and $y(t)$ apply:

$u_{\min} \leq u(t) \leq u_{\max}$ and $y_{\min} \leq y(t) \leq y_{\max}$ for $t \in [1,T]$.

The feedback controller

$$u(t) = C(z)(y^\circ(t) - y(t)) \qquad (1)$$

is parameterized according to the Youla parameterization, which, for a stable $P(z)$ as we assume here, boils down to an Internal Model Control (IMC) scheme, [12], where the controller is formed by a replica of the plant plus a tunable $Q(z)$ transfer function, see Figure 1.

According to the IMC parametrization, the transfer function $C(z)$ is given by

$$C(z) = \frac{Q(z)}{1-Q(z)P(z)} \qquad (2)$$

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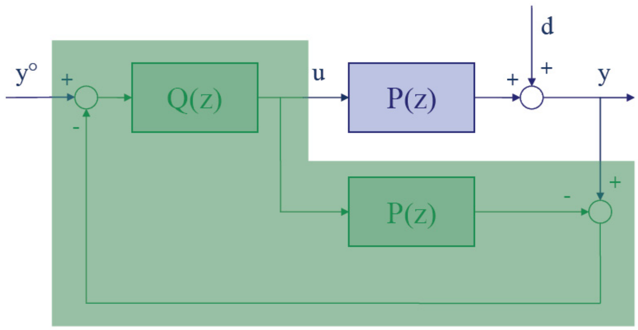

Fig. 1. The IMC scheme.

A key advantage of the IMC parametrization is that stability of the closed-loop system is simply obtained by letting $Q(z)$ vary over the set of stable transfer functions. In addition, the links from the disturbance $d(t)$ and the reference signal $y^\circ(t)$ to the system input and output are affine functions in $Q(z)$:

$$u(t) = Q(z)(y^\circ(t) - d(t)) \tag{3}$$

$$y(t) = P(z)Q(z)(y^\circ(t) - d(t)) + d(t). \tag{4}$$

In the sequel, we assume that $Q(z)$ is linearly parameterized in $\gamma = (\gamma_1, \dots, \gamma_k)$, i.e.,

$$Q(z) = \gamma_1 q_1(z) + \gamma_2 q_2(z) + \cdots + \gamma_k q_k(z), \tag{5}$$

where $q_i(z)$ are pre-specified stable transfer functions. When $k \to \infty$, for standard choices of the $q_i(z)$ such an expansion tends to cover the set of all transfer functions, while choosing a finite fixed k corresponds to limit oneself to a finite dimensional subspace.

By further elaborating the expressions in (3) and (4), $y(t)$ and $u(t)$ can be shown to be affine functions in γ obtained by appropriately filtering the disturbance realization $d(t)$ and the reference $y^\circ(t)$. Indeed, by plugging the expression for $Q(z)$ in (5), the input and the output of the controlled system (3) and (4) can be expressed as

$$u(t) = \big(\gamma_1 q_1(z) + \ldots + \gamma_k q_k(z)\big) v(t) \tag{6}$$

$$y(t) = P(z)\big(\gamma_1 q_1(z) + \ldots + \gamma_k q_k(z)\big) v(t) + d(t), \tag{7}$$

where $v(t) := y^\circ(t) - d(t)$. By introducing the vectors

$$\phi(t) = \begin{bmatrix} q_1(z)v(t) \\ q_2(z)v(t) \\ \vdots \\ q_k(z)v(t) \end{bmatrix} \qquad \psi(t) = \begin{bmatrix} P(z)q_1(z)v(t) \\ P(z)q_2(z)v(t) \\ \vdots \\ P(z)q_k(z)v(t) \end{bmatrix},$$

containing filtered versions of $v(t)$, (6) and (7) can be re-written as

$$u(t) = \phi(t)'\gamma$$
$$y(t) = \psi(t)'\gamma + d(t).$$

As a consequence, the 2-norm of the tracking error can be expressed as

$$\sum_{t=1}^{T} [y(t) - y^\circ(t)]^2 = \gamma' A \gamma + B\gamma + C,$$

where

$$A = \sum_{t=1}^{T} \psi(t)\psi(t)', \ B = -2\sum_{t=1}^{T} v(t)\psi(t)', \ C = \sum_{t=1}^{T} v(t)^2$$

are matrices that depend on $d(t)$ and $y^\circ(t)$ only.

Suppose that now one generates N disturbance realizations $d(t)_1, d(t)_2, \dots, d(t)_N$ from a model that describes the stochastic characteristics of noise. The following mathematical program can then be explicitly written, it is called a *scenario optimization* program:

$$\min_{\gamma, h \in \Re^{k+1}} h \text{ subject to:} \tag{8}$$
$$\gamma' A \gamma + B\gamma + C \le h$$
$$u_{\min} \le \phi(t)'\gamma \le u_{\max}, \ \forall t \in [1, T]$$
$$y_{\min} \le \psi(t)'\gamma + d(t) \le y_{\max}, \ \forall t \in [1, T]$$
$$\forall d(t) \in \{d(t)_i, \ i = 1, 2, \dots, N\}.$$

The scenario optimization (8) is a standard finite convex optimization program with quadratic and affine constraints. Consequently, a solution can be found at low computational cost via commercial solvers, e.g. the openly distributed `CVX`, [13], [14], or `YALMIP`, [15], even for relatively large N values.

Note that h represents an upper bound to the tracking error 2-norm $\sum_{t=1}^{T} [y(t) - y^\circ(t)]^2 = \gamma' A \gamma + B\gamma + C$ for any of the generated realizations of $d(t)$. Such an upper bound is minimized in (8) under the additional constraints that $u(t) = \phi(t)'\gamma$ and $y(t) = \psi(t)'\gamma + d(t)$ keep within the saturation/safety limits.

Let $\bar\gamma$ and $\bar h$ be the solution to the scenario optimization program (8). By construction, the performance $\bar h$ of the controller with parameter $\bar\gamma$ is guaranteed over the disturbance realizations $d(t)_1, d(t)_2, \dots d(t)_N$, and the bounds on the input and output signals are satisfied as well.

Moving one step ahead with respect to what has been so far described, we can further allow for some, say m, of the constraints to be violated. The goal is to improve the performance via the elimination of constraints and one can for instance remove constraints according to a greedy algorithm that at each step removes the constraint associated to the disturbance realization whose elimination leads to the largest immediate improvement in the tracking performance. Let $\gamma^\star$ and $h^\star (< \bar h)$ be the new scenario solution and cost value obtained after removing m constraints. The fundamental fact is that, if m and N are appropriately chosen, then the performance $h^\star$ is guaranteed to be achieved also for unseen disturbance realizations with probability $1 - \epsilon$. These results are made precise in the following theorem, which is obtained by applying recent results related to the randomized solution of semi-infinite chance-constrained programs, see [7].

Theorem 1: Select a 'violation parameter' $\epsilon \in (0, 1)$ and a 'confidence parameter' $\beta \in (0, 1)$. If m and N are such that

$$\binom{m+k}{m} \sum_{i=0}^{m+k} \binom{N}{i} \epsilon^i (1-\epsilon)^{N-i} \le \beta, \tag{9}$$

then, with probability (confidence) no smaller than $1-\beta$, the SBS-controller with parameter $\gamma^\star$ optimizes the control performance while satisfying the boundedness constraints for all disturbance realizations except for a violation set whose probability is at most ϵ. ■

Probability $1-\beta$ is called 'confidence' and demands some explanation. One should note that $\gamma^\star$ is a random quantity because it depends on the randomly generated constraints corresponding to $d(t)_1, \ldots, d(t)_N$. It may happen that the generated disturbance realizations are not representative enough of the other unseen disturbance realizations. In this case no generalization can be expected. Parameter β controls the probability for this to happen and the final result that the violation set associated with $\gamma^\star$ has probability no more than ϵ holds with confidence $1-\beta$. It is worth mentioning the fact that N and m given by (9) bear a logarithmic dependence on $1/\beta$ so that, for any practical purpose, β has very marginal importance: We can select β to be such a small number as 10^{-10} or even 10^{-20}, in practice zero, and still N and m do not grow significantly.

So far the SBS approach has been presented in this paper with reference to a tracking problem where the control performance is expressed in terms of 2-norm of the tracking error. The extension to constrained control problems with different cost functions is quite straightforward, the only condition being that convexity in γ must be retained.
In the next section, we show an example where the SBS approach is applied to an LQG control problem with input saturation constraints.

III. APPLICATION TO ACTIVE SUSPENSION CONTROL

Consider the two degrees of freedom active suspension model with an adjustable force element shown in Figure 2.

Under the assumption that the springs and dampers are linear, and that the tire damping is negligible, the active suspension can be described as a fourth order linear system in the state variables s_1, s_2, s_3 and s_4, representing the tire deflection from the equilibrium, the unsprung mass velocity, the suspension deflection from the equilibrium, and the sprung mass velocity, [16].

The evolution of the state vector $s = [s_1,\ s_2,\ s_3,\ s_4]'$ is governed by

$$\dot{s} = As + B\bar{f} + B_w n \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{m_{us}} & -\frac{r_{us}}{m_{us}} & \frac{k_s}{m_{us}} & \frac{r_s}{m_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{r_s}{m_s} & -\frac{k_s}{m_s} & -\frac{r_s}{m_s} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{m_s}{m_{us}} \\ 0 \\ -1 \end{bmatrix}, \quad B_w = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

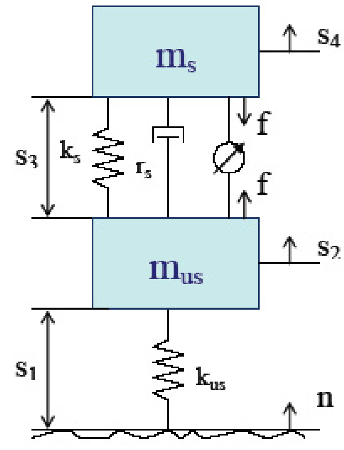

Fig. 2. The two degrees of freedom active suspension model.

whose parameters are the unsprung mass m_{us}, the sprung mass m_s, the suspension spring constant and damping coefficient k_s and r_s, and the tire stiffness k_{us}.
As for the input signals $\bar{f}$ and n, $\bar{f} = f/m_s$ represents the normalized adjustable force, whereas n is the road velocity disturbance. The disturbance n is described as a Gaussian noise with power spectral density $W = A_{road}v_s$ given by the product of the road roughness factor A_{road} and the vehicle forward velocity v_s, [18].

The goal is to regulate the adjustable force so as to minimize the quadratic cost

$$\frac{1}{t_f}\int_0^{t_f} \left(\rho_1 s_1^2(\tau) + \rho_3 s_3^2(\tau) + \dot{s}_4^2(\tau)\right) d\tau \tag{11}$$

where

$$\dot{s}_4(\tau) = \frac{r_s}{m_s}s_2(\tau) - \frac{k_s}{m_s}s_3(\tau) - \frac{r_s}{m_s}s_4(\tau) - \bar{f}(\tau),$$

subject to the input saturation constraint

$$|\bar{f}(\tau)| \leq u_{bound}, \quad \tau \in [0, t_f].$$

Cost (11) accounts for road holding and comfort through the tire deflection and sprung mass acceleration terms, whereas the suspension deflection term is introduced to avoid damaging the suspension. The contribution of these three terms is weighted through $\rho_1 > 0$ and $\rho_3 > 0$. The resulting cost is quadratic in the state and input variables.

State-feedback solutions like clipped LQG control have been proposed in the literature. However, measuring the tire deflection is quite difficult, which makes full state feedback solutions not applicable in practice. Variables that are easy to measure are the suspension deflection and the body acceleration. Here, we apply the SBS approach to design an output feedback controller based on the suspension deflection measurement. To this purpose, we shall refer to the discrete-time version of the problem with sampling time T_s, and denote the discrete time state vector, control input, and disturbance variables as $x(t)$, $u(t)$, and $w(t)$, respectively.

The disturbance $w(t)$ is a sequence of i.i.d. Gaussian random variables with zero mean and variance $\sigma_w^2 = \sqrt{\frac{2\pi W}{T_s}}$: $w(t) \sim WGN(0, \sigma_w^2)$.

We can describe the discrete time suspension system in terms of transfer functions as follows:

$$\begin{cases} x_1(t) = G_1(z)u(t) + H_1(z)w(t) \\ x_2(t) = G_2(z)u(t) + H_2(z)w(t) \\ x_3(t) = G_3(z)u(t) + H_3(z)w(t) \\ x_4(t) = G_4(z)u(t) + H_4(z)w(t) \end{cases} \tag{12}$$

If we take the suspension deflection as output

$$y(t) = x_3(t) + v(t), \quad v(t) \sim WGN(0, \sigma_v^2),$$

then, $P(z)$ appearing in the IMC scheme in Figure 1 is given by $G_3(z)$, whereas the disturbance $d(t)$ is the filtered version of the road disturbance $w(t)$ plus the noise measurement $v(t)$:

$$d(t) = H_3(z)w(t) + v(t).$$

The IMC control input is then given by

$$u(t) = -Q(z)d(t) = -Q(z)(H_3(z)w(t) + v(t))$$

and, by (12),

$$\begin{cases} x_1(t) = -G_1(z)Q(z)(H_3(z)w(t) + v(t)) + H_1(z)w(t) \\ x_2(t) = -G_2(z)Q(z)(H_3(z)w(t) + v(t)) + H_2(z)w(t) \\ x_3(t) = -G_3(z)Q(z)(H_3(z)w(t) + v(t)) + H_3(z)w(t) \\ x_4(t) = -G_4(z)Q(z)(H_3(z)w(t) + v(t)) + H_4(z)w(t) \end{cases}$$

This shows that the variables entering the control cost function as well as the input saturation constraint are affine in the parameter γ of $Q(z)$, with coefficients determined by appropriately filtered version of the disturbance realizations $w(t)$ and $v(t)$. As a consequence, the constrained optimization problem to be solved for determining γ according to the SBS approach is convex, and Theorem 1 holds for the resulting SBS controller.

The following values for the active suspension model parameters were adopted in the numerical examples, [17], [18]:

$$m_s = 320\ kg, \quad k_s = 18 \cdot 10^3\ \frac{N}{m}, \quad r_s = 10^3\ \frac{Ns}{m},$$
$$m_{us} = 40\ kg, \quad k_{us} = 200 \cdot 10^3\ \frac{N}{m},$$
$$A_{road} = 4.9 \cdot 10^{-6}\ m, \quad v_s = 88 \cdot 10^3\ \frac{m}{h}$$

The sampling time and the output measurement noise variance were set equal to $T_s = 10^{-2}\ s$ and $\sigma_v^2 = 10^{-9}\ m^2$.

The parameters entering the control problem formulation were: $t_f = 20$, $\rho_1 = 1100$, and $\rho_3 = 100$, [18], [16], whereas different values for the input saturation threshold u_{bound} were considered.

As for the functions $q_1(z), q_2(z), \dots, q_k(z)$ appearing in the parametrization $Q(z)$ of the IMC controller (see (5)), we chose

$$q_i(z) = Q_{LQG}(z)z^{-i+1},$$

where $Q_{LQG}(z)$ denotes the IMC parametrization of the controller obtained by solving the (unconstrained) infinite-horizon LQG output feedback control problem. The resulting $Q(z)$ transfer function is given by

$$Q(z) = Q_{LQG}(z)(\gamma_1 + \gamma_2 z^{-1} + \cdot + \gamma_k z^{-k+1}),$$

and reduces to $Q_{LQG}(z)$ when $\gamma_1 = 1$ and $\gamma_2 = \cdots = \gamma_k = 0$.

We next describe the numerical results obtained by applying the SBS approach. In this paper, we consider only the case when $m = 0$ (no constraint to be removed).

Fix $k = 3$ and $\beta = 10^{-5}$.

If we choose $\epsilon = 0.1$, then, the minimum value for N satisfying the bound (9) for $m = 0$ is $N = 159$.

By applying the SBS approach with $m = 0$ and $N = 159$ and $u_{bound} = 100$, we obtained $\gamma^\star = (0.982,\ -0.010,\ -0.021)$ and a performance $h^\star = 0.261$ guaranteed for all the disturbance realizations except for a set of probability not smaller than 0.9. In this case, $Q(z) \simeq Q_{LQG}(z)$ since u_{bound} is large compared to the values taken by the optimal LQG control input. We then reduced u_{bound} to $u_{bound} = 5$ and obtained $\gamma^\star = (0.319,\ 0.174,\ -0.155)$, $h^\star = 0.739$. Not surprisingly, the SBS controller is significantly different from the optimal (unconstrained) LQG controller.

Figures 3 and 4 represent the closed-loop system behavior when the SBS controllers computed for $u_{bound} = 100$ and $u_{bound} = 5$ are applied. Plots are obtained for the same realizations of the road velocity disturbance and measurement noise. In these figures, the tire deflection, suspension deflection, and body acceleration contributing to the control cost function are plotted together with the normalized adjustable force. Note that the normalized adjustable force is close to the saturation limits $\pm u_{bound}$ only in the case when $u_{bound} = 5$ (compare the corresponding plots of Figures 3 and 4). The sample performance is 0.255 ($u_{bound} = 100$) and 0.679 ($u_{bound} = 5$).

From Figure 3, it is clear that a saturation value of 5 applied to the output of the SBS controller designed for $u_{bound} = 100$ will result in a clipping of the normalized adjustable force control input (clipped LQG control). As for the SBS controller designed for $u_{bound} = 5$, applying a saturation value of 5 will not cause any clipping of the control input realization reported in Figure 4. The interpretation of this result is that the SBS approach guarantees that, if we run a sufficiently large number of Monte Carlo experiments, 90% of the control input realizations generated by the latter controller will not exceed $u_{bound} = 5$, and the one plotted in Figure 4 belongs to such a set of realizations.

IV. CONCLUSION

In this paper, we have described a randomized approach to constrained control design for linear systems affected by possibly unbounded stochastic disturbances. An example of application to active suspension control was presented. The

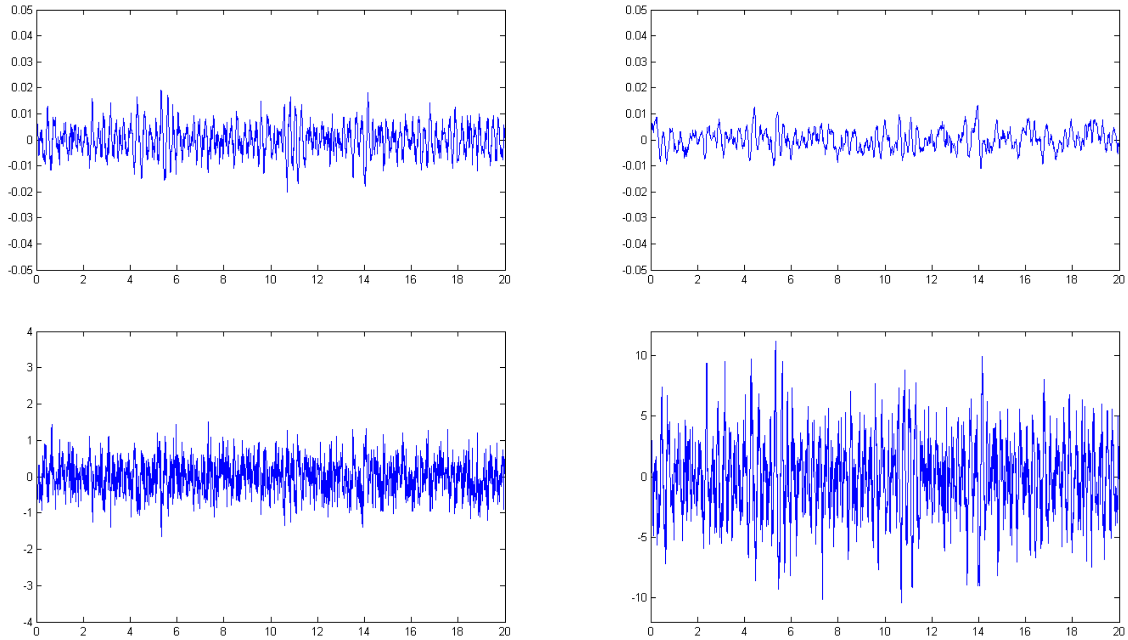

Fig. 3. Sample realizations of tire deflection (top left), suspension deflection (top right), body acceleration (bottom left), and normalized adjustable force (bottom right) for the active suspension with the SBS controller ($u_{bound} = 100$, $\epsilon = 0.1$, $m = 0$, $N = 159$).

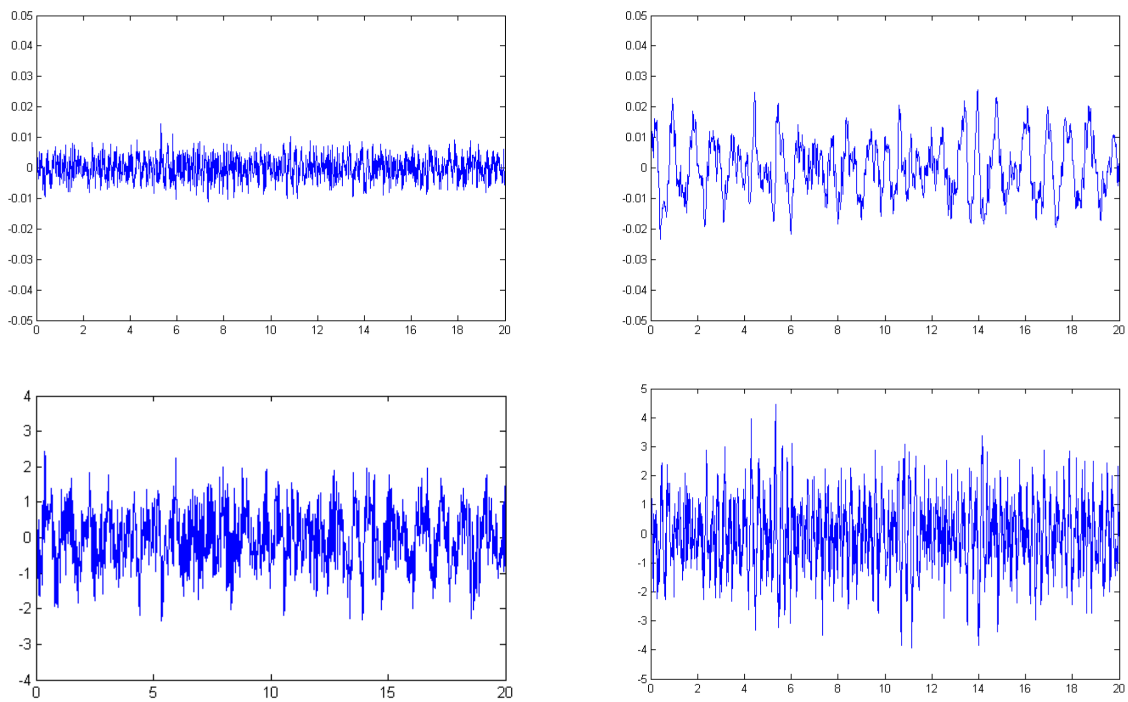

Fig. 4. Sample realizations of tire deflection (top left), suspension deflection (top right), body acceleration (bottom left), and normalized adjustable force (bottom right) for the active suspension with the SBS controller ($u_{bound} = 5$, $\epsilon = 0.1$, $m = 0$, $N = 159$).

proposed approach relies on a finite-dimensional parametrization of the IMC controller. The choice of this parametrization affects the achievable performance and deserves some consideration. Some guidelines can be found in the IMC related literature.

REFERENCES

[1] J. A. Primbs, C. H. Sung, *Stochastic receding horizon control of constrained linear systems with state and control multiplicative noise*, IEEE Trans. on Automatic Control 54(2), pp. 221-230, 2009.

[2] M. Cannon, B. Kouvaritakis, X. Wu, *Probabilistic constrained MPC*

for systems with multiplicative and additive stochastic uncertainty, IFAC World Congress, Seoul, Korea, 2008.
[3] M. Agarwal, E. Cinquemani, D. Chatterjee, J. Lygeros, *On convexity of stochastic optimization problems with constraints*, European Control Conference, Budapest, Hungary, 2009.
[4] G. Calafiore and M.C. Campi, *Uncertain convex programs: randomized solutions and confidence levels*, Math. Program., Ser. A vol. 102, pp. 25–46, 2005.
[5] G. Calafiore and M.C. Campi, *The scenario approach to robust control design*, IEEE Trans. on Automatic Control, vol. 51(5), pp. 742–753, 2006.
[6] M.C. Campi and S. Garatti, *The exact feasibility of randomized solutions of robust convex programs*, SIAM Journal on Control and Optimization, 19(3) pp. 1211–1230, 2008.
[7] M.C. Campi and S. Garatti, *Chance-constrained optimization via randomization: feasibility and optimality*, Available on Optimization Online, 2008.
[8] J.M. Maciejowski, *Predictive Control with Constraints, Prentice-Hall*, Pearson Education Limited, Harlow, UK, 2002.
[9] D.Q. Mayne, J.B. Rawlings, C.V. Rao, P. O. M. Scokaert, *Constrained model predictive control: Stability and optimality*, Automatica, vol. 36, pp. 789-814, 2000.
[10] P.O.M. Scokaert and D.Q. Mayne, *Min-max feedback model predictive control for constrained linear systems*, IEEE Trans. Automat. Contr., vol. 43, pp. 1136-1142, 1998.
[11] A. Bemporad, F. Borrelli, M. Morari, *Min-Max Control of Constrained Uncertain Discrete-Time Linear Systems*, IEEE Trans. on Automatic Control, vol. 48(9), pp. 1600–1606, 2003.
[12] M. Morari and E. Zafiriou, *Robust process control*. Prentice Hall, Englewood Cliffs, New Jersey, 1989.
[13] M. Grant and S. Boyd. *CVX: Matlab software for disciplined convex programming, version 1.21*. http://cvxr.com/cvx, April 2010.
[14] M. Grant and S. Boyd. *Graph implementations for nonsmooth convex programs*, In V. Blondel, S. Boyd, and H. Kimura, editors, Recent Advances in Learning and Con- trol, Lecture Notes in Control and Information Sciences, pp. 95-110. Springer-Verlag Limited, 2008.
[15] J. Lofberg. *YALMIP: A toolbox for modeling and optimization in MATLAB*, CACSD Conference, Taipei, Taiwan, 2004.
[16] N. Giorgietti, A. Bemporad, H. E. Tseng, D. Hrovat, *Hybrid Model Predictive Control Application Towards Optimal Semi-Active Suspension*, IEEE ISIE 2005, Dubrovnik, Croatia, 2005.
[17] T. Gordon, C. Marsh, and M. Milsted, *A comparison of adaptive LQG and nonlinear controllers for vehicle suspension systems*, Vehicle System Dynamics, vol. 20, pp. 321–340, 1991.
[18] D. Hrovat, *Survey of advanced suspension developments and related optimal control applications*, Automatica, 33(10), pp. 1781-1817, 1997.